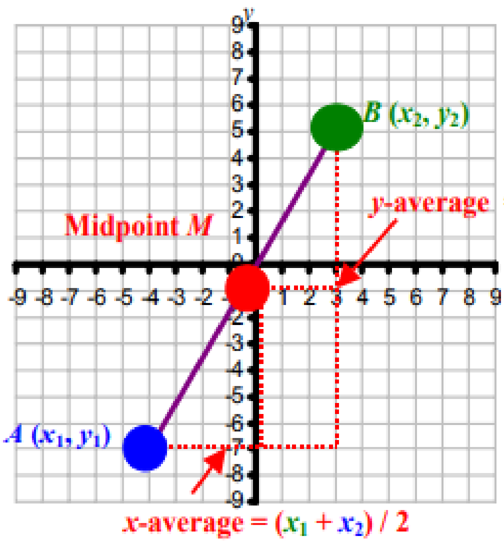
Ratu Navula College

Year 12 Mathematics Lesson Notes – Week 2

Strand 4: Coordinate Geometry **Sub Strand 4.1: Application of Coordinate Geometry**

Lesson 45: Midpoint

Learning Outcome: Find the midpoint of a line.



Midpoint: - the location (coordinate) of a point in the middle of a line segment.
- the length of one side of the midpoint is equivalent to the length of the other side.

Midpoint of a Line Segment

M = (average of the *x*-coordinates, average of the *y*-coordinates)

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 1

Find the midpoint of the line joining the points P (– 2, 3) and Q (2 , 5).

Let

$\boldsymbol{x_1 = -2}$ M.P $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\boldsymbol{x_2 = 2}$ $= \left(\frac{-2 + 2}{2}, \frac{3 + 5}{2}\right)$

$\boldsymbol{y_1 = 3}$ $= \left(\frac{0}{2} + \frac{8}{2}\right)$

$\boldsymbol{y_2 = 5}$ $= (\mathbf{0}, \mathbf{4})$

Example 2

$\overline{AB}$ is the diameter of a circle with the centre at (2 ,0). If A is at (-3, 2), then the coordinates of B are

A. (7, -2)

B. (-7, 2)

C. (-2, 7)

D. (7 , 2)

M.P $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

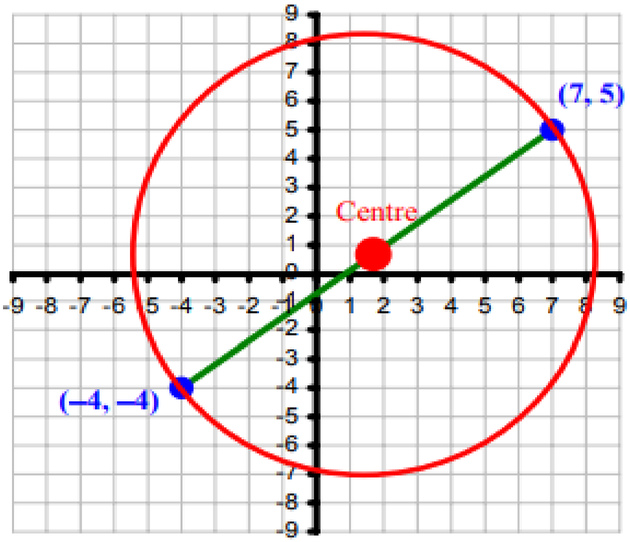
(2,0) $= \left(\frac{-3+x_2}{2}, \frac{2+y_2}{2}\right)$

$2 = \left(\frac{-3+x_2}{2}\right)$ and $0 = \left(\frac{2+y_2}{2}\right)$

$4 = -3 + x_2$ and $0 = 2 + y_2$

$7 = x_2$ and $-2 = y_2$

Example 3: A diameter of a circle has endpoints (7, 5) and (–4, –4). Find the coordinate of the centre.



$$M = \left(\frac{x_2 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{7 + -4}{2}, \frac{5 + -4}{2}\right)$$

$$Centre = \left(\frac{3}{2}, \frac{1}{2}\right)$$

Class Activity 45

1. Use the midpoint formula to find the coordinates of the midpoint of the line segment joining the following pairs of points.
   a) (−5, 1), (−1, −8)
   b) (4, 2), (11, −2)

2. The coordinates of the midpoint, M, of the line segment AB are (2, −3). If the coordinates of A are (7, 4), find the coordinates of B.

3. The vertices of a triangle are A (2, 5), B 11, −3) and C (−4, 3). Find:
   a) the coordinates of P, the midpoint of AC
   b) the coordinates of Q, the midpoint of AB

Strand 4: Coordinate Geometry **Sub Strand 4.1: Application of Coordinate Geometry**

Lesson 46: Distance Between Two Points

Learning Outcome: Find the distance between two points.

If A (x_1, y_1) and B (x_2, y_2) are two points then the distance between these two points is found by the formula given below.

$$\mathbf{d} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Steps:

1. Use the distance formula
2. Label the two points as x_2, x_1, y_2, y_1
3. Put the values into the formula
4. Use the order of operation and simplify.

Example 1

Find the length of the line segment whose end points are (5,4) and (-3,4).

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3-5)^2 + (4-4)^2} \\ &= \sqrt{(-8)}^2 \\ &= \sqrt{64} \\ &= 8\ units \end{aligned}$$

Example 2

Find the distance between the points P (−1, 5) and Q (3, −2).

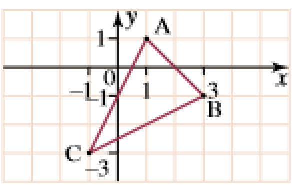
THINK	WRITE
1 Let P have coordinates (x_1, y_1).	Let $(x_1, y_1) = (-1, 5)$
2 Let Q have coordinates (x_2, y_2).	Let $(x_2, y_2) = (3, -2)$
3 Find the length PQ by applying the formula for the distance between two points.	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{[3 - (-1)]^2 + (-2 - 5)^2}$ $= \sqrt{(4)^2 + (-7)^2}$ $= \sqrt{16 + 49}$ $= \sqrt{65}$ $= 8.06$ (correct to 2 decimal places)

How to determine the distance if three or more points are given

1. Label the points using A,B and C
2. Find the 3 lengths (AB,AC,BC) between the points using the distance formula
   Note: if the length cannot be squared rooted evenly, leave in radical form.
3. If it is a right angled triangle then apply Pythagoras theorem $(a^2 + b^2 = c^2)$ to prove if it's a right angled triangle or not

Example 1

Prove that the points A (1, 1), B (3, −1) and C (−1, −3) are the vertices of an isosceles triangle.

THINK	WRITE/DRAW
1 Plot the points and draw the triangle. *Note*: For triangle ABC to be isosceles, two sides must have the same magnitude.	
2 AC and BC seem to be equal. Find the length AC. A $(1, 1) = (x_2, y_2)$ C $(-1, -3) = (x_1, y_1)$	$AC = \sqrt{[1 - (-1)]^2 + [1 - (-3)]^2}$ $= \sqrt{(2)^2 + (4)^2}$ $= \sqrt{20}$ $= 2\sqrt{5}$

3 Find the length BC.
B $(3, -1) = (x_2, y_2)$
C $(-1, -3) = (x_1, y_1)$

$$BC = \sqrt{[3-(-1)]^2 + [-1-(-3)]^2}$$
$$= \sqrt{(4)^2 + (2)^2}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

4 Find the length AB.
A $(1, 1) = (x_1, y_1)$
B $(3, -1) = (x_2, y_2)$

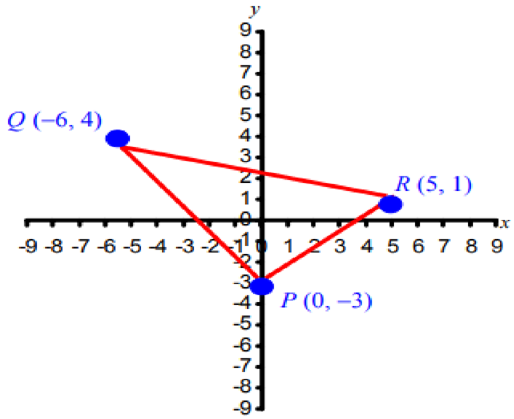
$$AB = \sqrt{[3-(1)]^2 + [-1-(1)]^2}$$
$$= \sqrt{(2)^2 + (-2)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

5 State your proof.

Since AC = BC ≠ AB, triangle ABC is an isosceles triangle.

Example 2

A triangle has vertices at P (0,-3), Q(-6,4) and R (5,1). Find the perimeter of the triangle to the nearest tenth of a unit and classify it



For the perimeter of ΔPQR, we must find the distances of $\overline{PQ}$, $\overline{QR}$ and $\overline{RP}$.

$$d_{\overline{PQ}} = \sqrt{(-6-0)^2 + (4--3)^2}$$
$$= \sqrt{(-6)^2 + 7^2} = \sqrt{36+49}$$
$$d_{\overline{PQ}} = \sqrt{85}$$

$$d_{\overline{QR}} = \sqrt{(5--6)^2 + (1-4)^2}$$
$$= \sqrt{11^2 + (-3)^2} = \sqrt{121+9}$$
$$d_{\overline{QR}} = \sqrt{130}$$

$$d_{\overline{RP}} = \sqrt{(0-5)^2 + (-3-1)^2}$$
$$= \sqrt{(-5)^2 + (-4)^2} = \sqrt{25+16}$$
$$d_{\overline{RP}} = \sqrt{41}$$

$$Perimeter = \sqrt{85} + \sqrt{130} + \sqrt{41}$$

$Perimeter \approx 27.0$ units

Since all three sides of the triangle are different in length, it is a SCALENE TRIANGLE.

Class Activity 46

1. Find the distance between the following points.
   i. (2,5) and (6,8)
   ii. (-1,3) and (-7,-5)
   iii. (4, -5) and (1,1)
   iv. (1,7) and (1,-6)

2. Prove that the points A (0, −3), B (−2, −1) and C (4, 3) are the vertices of an isosceles triangle.

Strand 4: Coordinate Geometry **Sub Strand 4.1: Application of Coordinate Geometry**

Lesson 47: Gradient Of A Straight Line

Learning Outcome: Find the midpoint of a line.

Let A (x_1, y_1) and B (x_2, y_2) be any two points on a straight line. The **slope** or **gradient** of the line AB can be calculated by the formula given below.

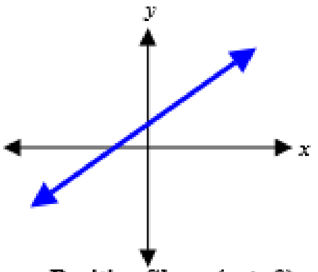
$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad or \quad m = \tan\theta$$

$m \rightarrow gradient$

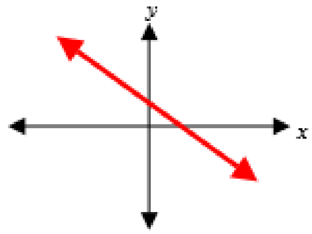
Note:

- θ is the **angle** that the **line makes with the positive** $x-$**axis.**

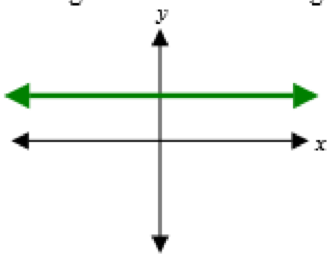
In general, slopes can be classified as follows:



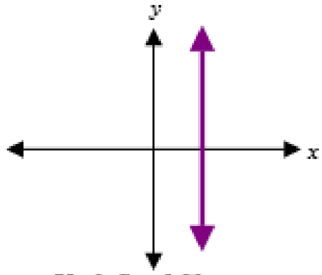
Positive Slope ($m > 0$)
Line goes UP from left to right.



Negative Slope ($m < 0$)
Line goes DOWN from left to right.



Zero Slope ($m = 0$)
Horizontal (Flat) Line [Rise = 0]



Undefined Slope
Vertical Line [Run = 0]

Example 2

Find the gradient of the straight line passing through the following points.

1 (1, 3) and (4, 7)

2 (6, −2) and (2, −1)

Solutions

1 Let (x_1, y_1) be (1, 3) and (x_2, y_2) be (4, 7).

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{4-1} = \frac{4}{3}$$

$\therefore$ The gradient is $1\frac{1}{3}$.

2 Let (x_1, y_1) be (6, −2) and (x_2, y_2) be (2, −1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-2)}{2-6} = \frac{1}{-4}$$

$\therefore$ The gradient is $-\frac{1}{4}$.

Example 3

If the slope of a line is $-\frac{2}{3}$ and it passes through (4, 5) and (-8, p). Find the value of p?

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$\frac{-2}{3}=\frac{p-5}{-8-4}$$
$$\frac{-2}{3}=\frac{(p-5)}{-12}$$

$$-2(-12)=3(p-5)$$
$$24=3p-15$$
$$24+15=3p$$
$$39=3p$$
$$\frac{39}{3}=p$$

$$p=13$$

Example 4

What angle does the line joining the points A$(-2, 1)$ and B$(2 , 4)$ make with the positive $x-$axis?

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{4-1}{2--2}$$
$$=\frac{3}{4}$$

Gradients is also $m = \tan\theta$

$$\frac{3}{4}=\tan\theta$$

$$\theta=\tan^{-1}\frac{3}{4}$$
$$=36.87°$$

Class Activity 47

1. Use the formula $m=\frac{y_2-y_1}{x_2-x_1}$ to find the gradient of the straight line passing through the points:

   a) (2, 6) and (5, 7)

   b) (4, 2) and (5, 6)

   c) (0, 0) and (5, 2)

Strand 4: Coordinate Geometry **Sub Strand 4.1: Application of Coordinate Geometry**

Lesson 48: Equation Of A Straight Line

Learning Outcome: Calculate the equation of a line

General Form :

$y = mx + c$ **or** $ax + by + c = 0$

gradient (m) y-intercept (c)

Gradient / Intercept Method

m is gradient and c is the y – intercept

↓

$y = mx + c$

Gradient and a Point Method

point (x_1, y_1) with gradient m

↓

$y - y_1 = m(x - x_1)$

Two Points Method

Two points (x_1, y_1) and (x_2, y_2)

↓

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1

Write down the gradient and y-intercepts of these lines.

a $y = 4x - 2$

$m = 4$ $b = -2$

Gradient $= 4$

y-intercept $= -2$

b $y = 3 - 5x$

$b = 3$ $m = -5$

Gradient $= -5$

y-intercept $= 3$

c $2x + 3y = 12$

$3y = -2x + 12$

$y = -\frac{2}{3}x + 4$

$m = -\frac{2}{3}$ $b = 4$

Gradient $= -\frac{2}{3}$

y-intercept $= 4$

Example 2

Find the equation of the line that passes through the points (−1, 2) and (2, 8).

Solution

Let the equation of the line be:

$y = mx + c$

Now $m = \frac{y_2 - y_1}{x_2 - x_1}$

$(x_1, y_1) = (-1, 2)$
$(x_2, y_2) = (2, 8)$

$= \frac{8-2}{2-(-1)}$

$= \frac{6}{3}$

$\therefore m = 2$

$\therefore y = 2x + c$ (since $m = 2$)

(2, 8) lies on the line.

$\therefore 8 = 2(2) + c$

$\therefore c = 4$

$\therefore$ The equation is $y = 2x + 4$.

or

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

(x_1, y_1) is (−1, 2), (x_2, y_2) is (2, 8)

$\therefore y - 2 = \frac{8-2}{2-(-1)}[x - (-1)]$

$y - 2 = \frac{6}{3}(x + 1)$

$y - 2 = 2(x + 1)$

$y - 2 = 2x + 2$

$\therefore y = 2x + 4$ is the equation of the line.

Example 3

1 Find the equation of the line that passes through (1, 4) and has gradient 2.

2 A straight line has gradient $-\frac{1}{2}$ and passes through the point (1, 3). Find the equation of this line.

$y - y_1 = m(x - x_1)$
or
$y = mx + c$

Solutions

1 Let the equation of the line be:

$y = mx + c$

$\therefore y = 2x + c$ ($m = 2$ is given)

$4 = 2(1) + c$ [(1, 4) lies on the line]

$4 = 2 + c$

$\therefore c = 2$

$\therefore$ The equation is $y = 2x + 2$.

or 1 $y - y_1 = m(x - x_1)$

(x_1, y_1) is (1, 4), $m = 2$

$\therefore y - 4 = 2(x - 1)$

$y - 4 = 2x - 2$

$\therefore y = 2x + 2$ is the equation of the line.

2 Let the equation be:

$y = mx + c$

$\therefore y = -\frac{1}{2}x + c$ ($m = -\frac{1}{2}$ is given)

$3 = -\frac{1}{2}(1) + c$ [(1, 3) is on the line]

$3 = -\frac{1}{2} + c$

$\therefore c = 3\frac{1}{2}$

$\therefore$ The equation is $y = -\frac{1}{2}x + 3\frac{1}{2}$.

or 2 $y - y_1 = m(x - x_1)$

(x_1, y_1) is (1, 3), $m = -\frac{1}{2}$

$\therefore y - 3 = -\frac{1}{2}(x - 1)$

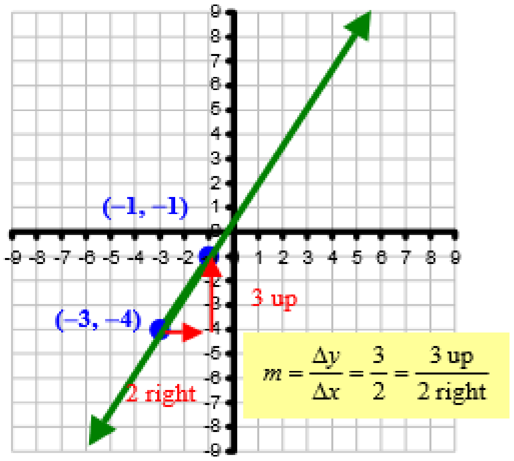
$y - 3 = -\frac{1}{2}x + \frac{1}{2}$

$\therefore y = -\frac{1}{2}x + 3\frac{1}{2}$ is the equation of the line.

Example 4

Find the equation in a point- slope form and standard form given the following

a. $(-3, -4)$ and $m = \frac{3}{2}$



For Slope-Point form:

$y - y_1 = m(x - x_1)$

$y - (-3) = \frac{3}{2}(x - (-4))$

$y + 3 = \frac{3}{2}(x + 4)$

For Standard form:

$y + 3 = \frac{3}{2}(x + 4)$

$2(y + 3) = 3(x + 4)$

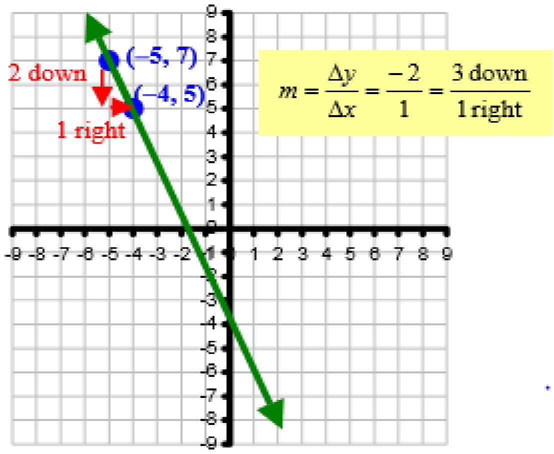
$2y + 6 = 3x + 12$

$0 = 3x - 2y + 12 - 6$

$\mathbf{0 = 3x - 2y - 6}$

Bringing all the terms to the right-hand side of equation will ensure a positive coefficient for the x term.

b. $(-5, 7)$ and $m = -2$



For Slope-Point form:

$y - y_1 = m(x - x_1)$

$y - 7 = -2(x - (-5))$

$y - 7 = -2(x + 5)$

For Standard form:

$y - 7 = -2(x + 5)$

$y - 7 = -2x - 10$

$2x + y - 7 + 10 = 0$

$\mathbf{2x + y + 3 = 0}$

Bringing all the terms to the left-hand side of equation will ensure a positive coefficient for the x term.

Class Activity 48

1. Write the gradient and the y-intercept of the following lines:
   a) $y = 4x - 5$
   b) $y = 9 - x$
   c) $2y = 3x - 5$

2. Find the equation of the straight line (giving your answer in the form $y = mx + c$)
   a) gradient 2 and passes through the point (1, 3)
   b) slope 4 and passes through the point (−1, 6)

3. A straight line has a gradient of 2 and passes through the point (3, 2). Find the equation of the line.

4. Find the equation of the line that passes through the points (−2, −2) and (1, 4).

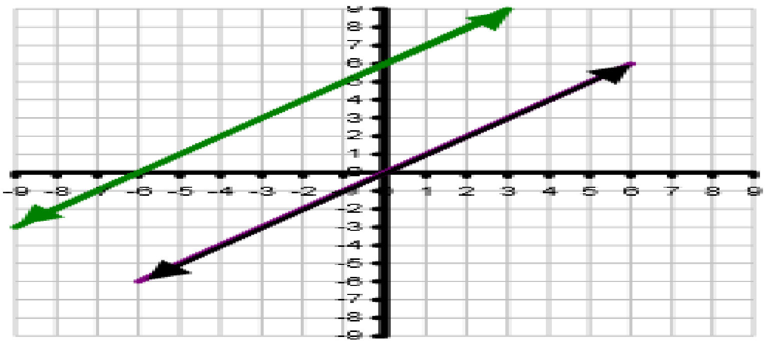
Strand 4: Coordinate Geometry **Sub Strand 4.1: Application of Coordinate Geometry**

Lesson 49: Parallel Lines

Learning Outcome: able to identify the parallel line and find the gradient of the parallel line.

Two or more lines are considered to be **parallel** when they satisfy the three conditions given below.

- Their **gradients are equal**. This means the gradient of the first line m_1 is equal to the gradient of the second line m_2.
- They do not intersect at any point.
- They are equidistant apart.



Parallel Lines
slope of line 1 = slope of line 2

$$m_{l_1} = m_{l_2}$$

Example 1

Show that AB is parallel to CD given that A has coordinates (−1, −5), B has coordinates (5, 7), C has coordinates (−3, 1) and D has coordinates (4, 15).

THINK

1. Find the gradient of AB by applying the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

WRITE

Let A $(-1, -5) = (x_1, y_1)$ and B $(5, 7) = (x_2, y_2)$

Since $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m_{AB} = \frac{7 - (-5)}{5 - (-1)}$$
$$= \frac{12}{6}$$
$$= 2$$

2. Find the gradient of CD.

Let C $(-3, 1) = (x_1, y_1)$ and D $(4, 15) = (x_2, y_2)$

$$m_{CD} = \frac{15 - 1}{4 - (-3)}$$
$$= \frac{14}{7}$$
$$= 2$$

3. Draw a conclusion. (*Note*: || means 'is parallel to'.)

Since $m_{AB} = m_{CD} = 2$, then AB || CD.

Example 2

Find the equation of the line that passes through the point (1,4) and is parallel to $y = 3x + 2$

Let the equation of the line be $y = mx + c$.

$y = 3x - 2$ has gradient 3

$\therefore\ m = 3$ (Parallel lines have equal gradients.)

$\therefore\ y = 3x + c$

$4 = 3(1) + c$, [(1, 4) lies on the line]

$\therefore\ c = 1$

$\therefore$ The equation of the line is $y = 3x + 1$.

Class Activity 49

1. Are the following pairs of lines parallel or not?
   a) $y = 3x + 2 \ and \ y = 3x - 1$
   b) $y = 5x - 2 \ and \ y = 2x - 5$
   c) $y = 2x + 1 \ and \ 2x - y + 3 = 0$

2. Find the equation of the line that has y- intercept 3 and is parallel to $y = 5x - 1$

3. Find the equation of the line that passes through the point $(4, -1)$ and is parallel to the line with equation $y = 2x - 5$

4. Find the equation of the straight line that passes through the point $(4, -3)$ and parallel to $3y + 2x = -3$.