RATU NAVULA COLLEGE

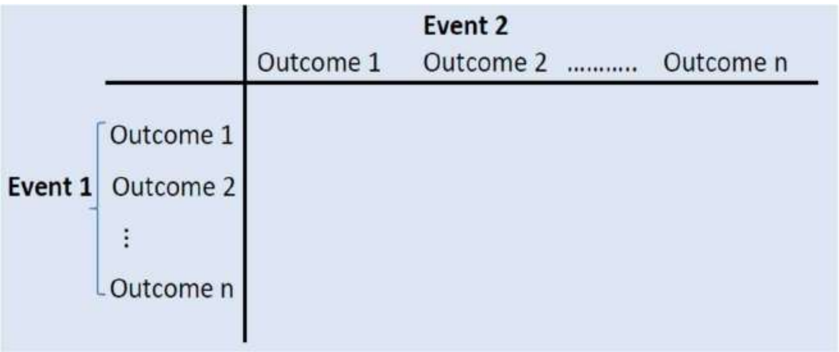
YEAR 12 Mathematics Lesson Notes – Week 8

Strand 7: Statistics **Sub Strand 7.1: Statistical Analysis**

Lesson 75: Lattice Diagram

Learning Outcome: Use lattice diagram to find probabilities.

Lattice diagrams are used to display the outcomes of two-stage events. This is much easier than tree diagrams.

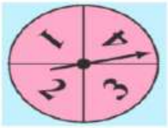


Example 1

A spinner with numbers 1 to 4 is spun an unbiased coin is tossed. Draw a lattice diagram and use it to give the following probabilities:

(a) P(head and a 4)

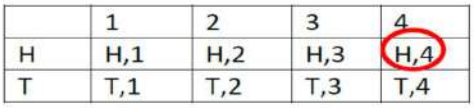
(b) P(head *or* a 4)



Spinner

Coin

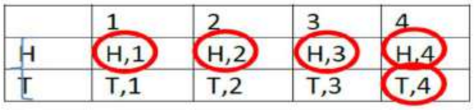
Answers:



	1	2	3	4
H	H,1	H,2	H,3	H,4
T	T,1	T,2	T,3	T,4

Total outcome = 8

$P(\text{head and a 4}) = {}^1\!/_8$

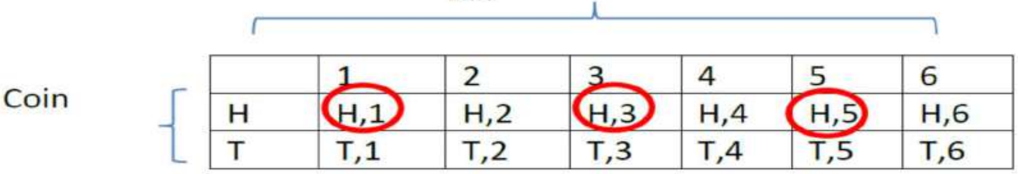


	1	2	3	4
H	H,1	H,2	H,3	H,4
T	T,1	T,2	T,3	T,4

$P(\text{head or a 4}) = {}^5\!/_8$

Example 2

A die is rolled and a coin is tossed simultaneously. Find the probability of getting a head and an odd number.



	1	2	3	4	5	6
H	H,1	H,2	H,3	H,4	H,5	H,6
T	T,1	T,2	T,3	T,4	T,5	T,6

Total outcome = 12

$$P(\text{head and odd number}) = \frac{3}{12} = 0.25 \text{ or } {}^1\!/_4$$

Activity

1. Two dice are rolled and the numbers on the **uppermost faces** are observed. The sample space is shown in the **lattice diagram** below.

Die 2

		1	2	3	4	5	6
Die 1	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

What is the probability of observing

(a) a sum of 8?

(b) an even number on die 2?

2. A coin and a die are tossed at the same time.

(a) How many possible outcomes are there in the sample space?

(b) Find the probability of getting a tail on the coin and an even number on the die?

Strand 7: Statistics **Sub Strand 7.1: Statistical Analysis**

Lesson 76: Probability Without Replacement

Learning Outcome: Calculate probabilities without replacement.

Probability without replacement or dependent probability is where you do not put the object back so that the number is less than for the first draw. The probabilities for the 2 trials will not be the same, hence the tree diagram will be more appropriate to use.

Example 1

A box contains 2 green, 1 blue and 2 yellow balls, all of the same shape and size. A ball is picked at random from the box. What is the probability that the ball picked will be:

i) green?

ii) not yellow?

If the first ball is picked and without replacing a second one is picked from the box, what is the probability that:

iii) the first ball picked is blue and the second is yellow?

iv) both balls are of the same color?

S = {G, B, Y}

2 1 2 ➲ Total is 5

$P(G) = \frac{2}{5}$, $P(B) = \frac{1}{5}$, $P(Y) = \frac{2}{5}$

Tree Diagram :

i) green

$P(G) = \frac{2}{5}$ or 0.4

ii) not yellow: do not include Y

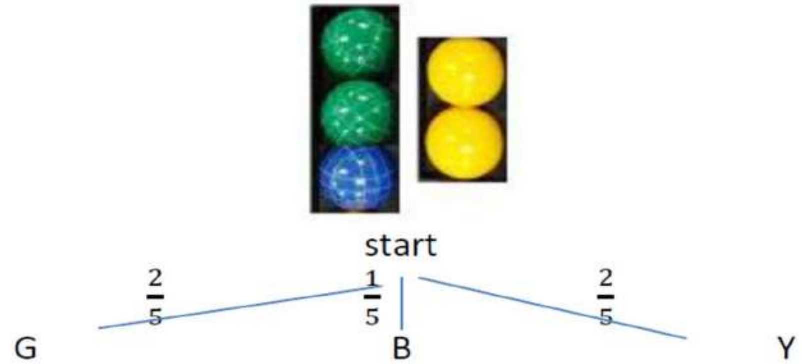
P(not Y) means P(B & G) =

$\frac{2+1}{5} = \frac{3}{5}$ or 0.6

Another method:

$1 - P(Y) = 1 - \frac{2}{5} = \frac{3}{5}$ or 0.6

Tree Diagram :

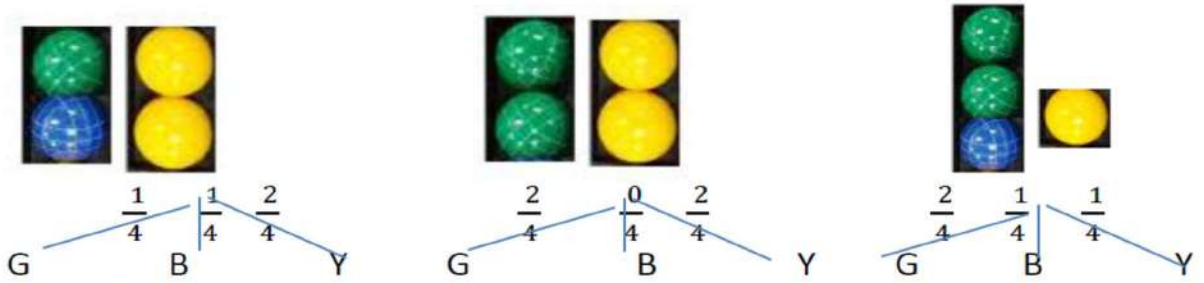


first trial

If green is picked, then G number decreases only

If blue is picked, then B number decreases only

If yellow is picked, then Y number decreases only



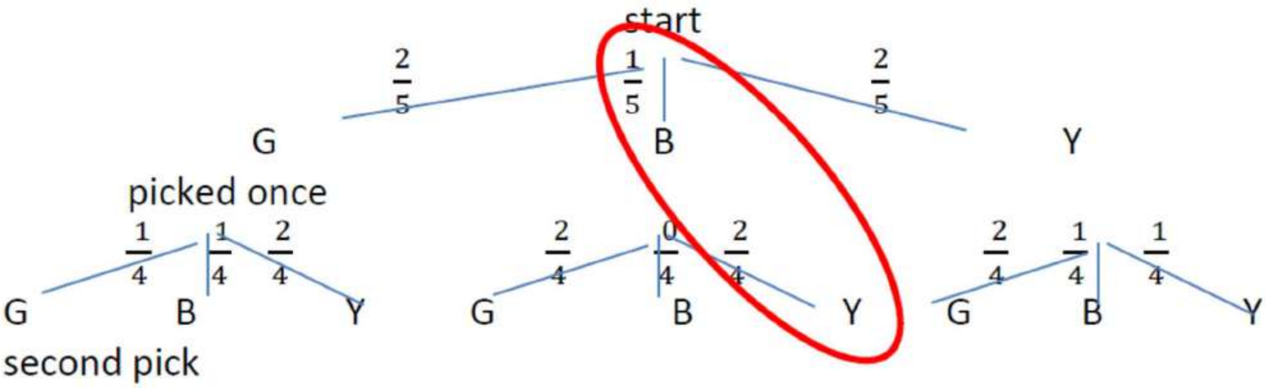
second trial

Outcomes:

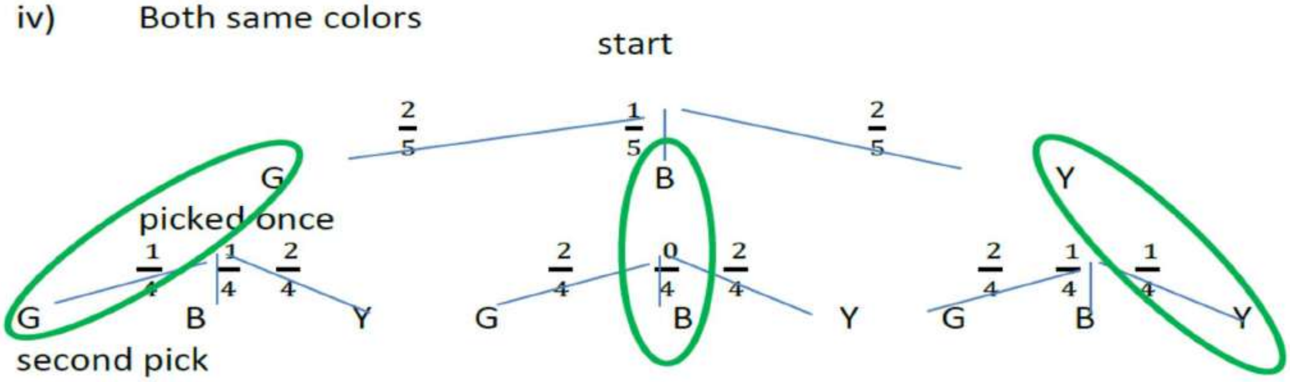
GG GB GY BG BB BY YG YG YY

iii) the first is blue second is yellow

$$P(BY) = P(B) \times P(Y) = \frac{1}{5} \times \frac{2}{4} = \frac{1}{10} or 0.1$$



iv) Both same colors



$$P(same) = P(GG) + P(BB) + P(YY)$$

$$= P(G) \times P(G) + P(B) \times P(B) + P(Y) \times P(Y)$$

$$= \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{0}{4} + \frac{2}{5} \times \frac{1}{4}$$

$$= \frac{1}{10} + 0 + \frac{1}{10} = \frac{1}{5} or 0.2$$

Example 2

A jar consists of 21 sweets. 12 are green and 9 are blue. Noa picked two sweets at random.

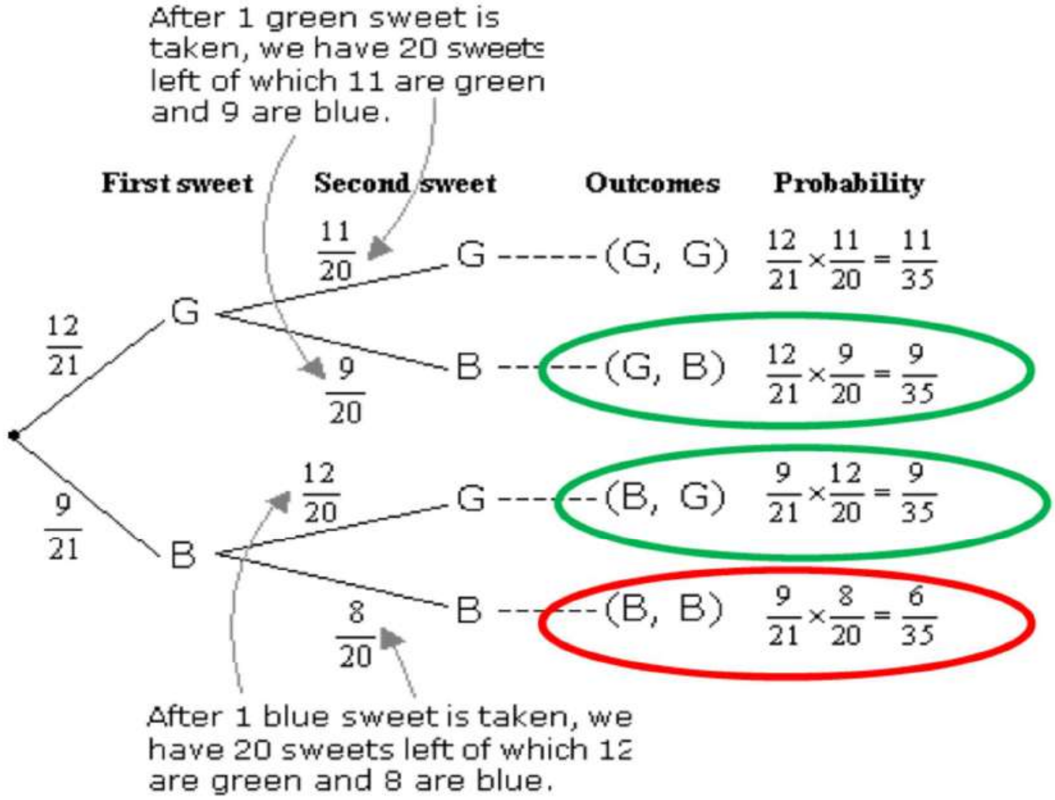
a) Draw a tree diagram to represent the experiment.

b) Find the probability that

   i) both sweets are blue.

   ii) One sweet is blue and one sweet is green.

a) Although both sweets were taken together it is similar to picking one sweet and then the second sweet without replacing the first sweet.


After 1 green sweet is taken, we have 20 sweets left of which 11 are green and 9 are blue.

After 1 blue sweet is taken, we have 20 sweets left of which 12 are green and 8 are blue.

b)

i) P(both sweets are blue)

P(B B)

$$P(BB)=P(B)\times P(B)$$
$$=\frac{9}{21}\times\frac{8}{20}=\frac{6}{35}or0.1714$$

ii) P(one sweet is blue and one is green)

P(G, B) or P(B, G)

$$P(GB)+P(BG)$$
$$=P(G)\times P(B)+P(B)\times P(G)$$
$$=\frac{12}{21}\times\frac{9}{20}+\frac{9}{21}\times\frac{12}{20}$$
$$=\frac{9}{35}+\frac{9}{35}=\frac{18}{35}or0.5143$$

Lesson 77: Probability Without Replacement

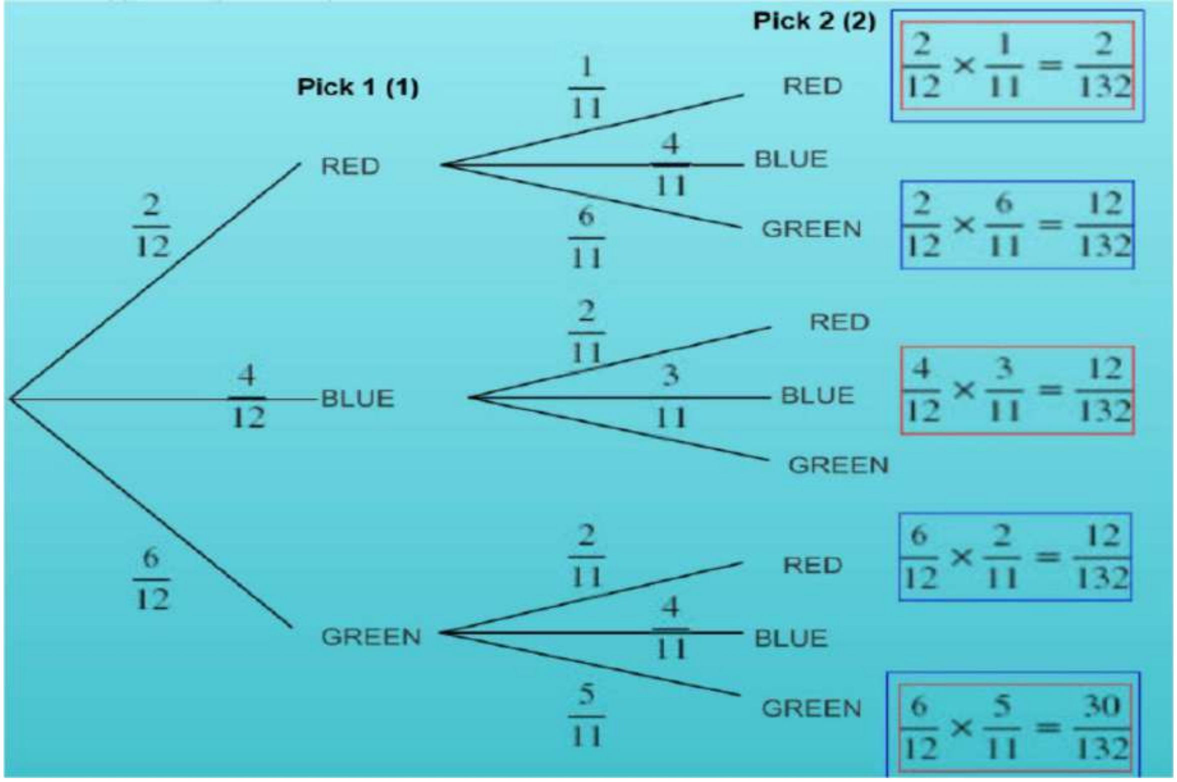
Learning Outcome: Calculate probabilities without replacement.

Example 3

We now have a bag with 12 marbles (2 red, 4 blue, 6 green). We have to pick twice (not replacing the 1st marble) Find:

a) P (Same Colour Twice)

b) P (Not Blue)



a) To find the answer to part a we have to look at all the possibilities where we get the same colour twice: RED & RED, BLUE & BLUE and GREEN & GREEN. We then have to calculate the probabilities for these combined events. Lastly, we have to add these probabilities. The solution is P (Same Colour Twice) = 1/3

$$\frac{2}{132}+\frac{12}{132}+\frac{30}{132}=\frac{44}{132}=\frac{1}{3}$$

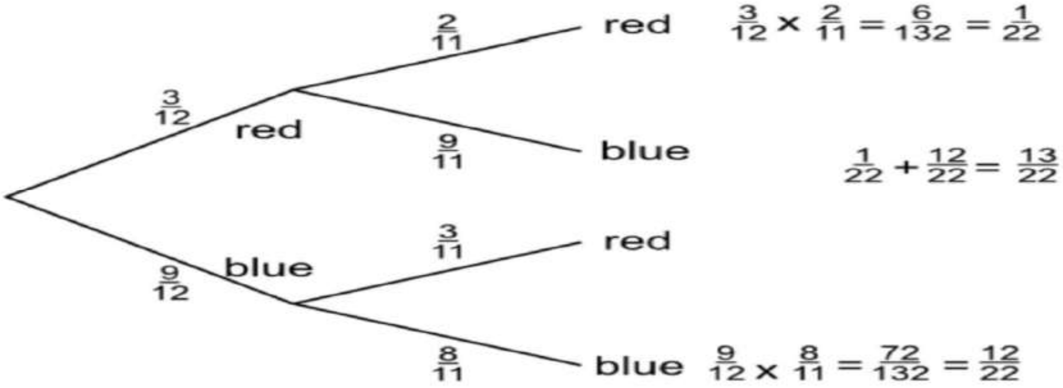
b) We need to find all the possibilities that do not contain blue. These are: • RR • RG • GR • GG We need to find the probability of all these combined events. Now we simply need to add all these combined probabilities together. The solution is P (Not Blue) = 14/33

$$\frac{2}{132}+\frac{12}{132}+\frac{12}{132}+\frac{30}{132}=\frac{56}{132}=\frac{14}{33}$$

Example 4

A bag contains 12 balls. 3 are red and 9 are blue. A ball is drawn at random, and NOT replaced in the bag. By drawing a probability tree, or otherwise, calculate the probability of drawing two consecutive balls of the same colour.

Note that the balls are NOT replaced: therefore the denominator of the fractions changes between the first draw and the second draw.



Answer: Probability of two reds or two blues:

$(\mathbf{P}(\text{red}_1) \times \mathbf{P}(\textbf{red})_2) + (\mathbf{P}(\text{blue})_1 \times \mathbf{P}(\text{blue})_2)$

$$\frac{3}{12} \times \frac{2}{11} + \frac{9}{12} \times \frac{8}{11} = \frac{13}{22}$$

Activity

1. A jar contains 5 red marbles and 3 green marbles. Two marbles are drawn in succession form the jar **without** replacement. What is the probability that one of each colour marble is drawn?

2. A jar contains 3 red marbles and 2 green marbles, all of same size and shape. A marble is withdrawn at random and its colour is noted. **Without** replacing this marble, another marble is randomly withdrawn. What is the probability that the marbles are of different colours?

Strand 7: Statistics **Sub Strand 7.1: Statistical Analysis**

Lesson 78: Venn Diagrams

Learning Outcome: Find probabilities using Venn Diagrams.

A Venn diagram is constructed with a collection of simple closed curves drawn in a plane as well as overlapping circles.

Properties of Venn diagram:

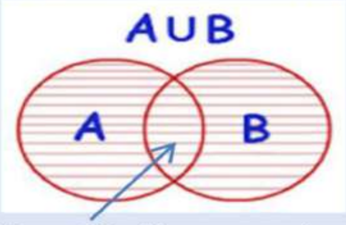
- The interior of the circle symbolically represents the elements of the set (A) while the exterior represents elements that are not members of the set (A' or A^c). This is also referred to as **Compliment of A**.

$P(A) + P(A') = 1$

$P(A) = 1 - P(A^c)$

$P(A^c) = 1 - P(A)$

- **Empty Set** is a set with no elements common. Shown by { } or Ø.
  A$\cap B = \emptyset$ *or* { } null set
- Two sets, A and B, is said to be **disjoint** if A$\cap B = \emptyset$
- "**Union**" of sets has the special symbol ∪ that means it is the set containing all the elements in the first or in the second set.



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- $P(A \cap B)$: "Intersection" of a set has special symbol ∩ and it must be in both sets, A and B.

Example 1

If set A = {1, 3, 5, 7, 9} and set B = {2, 3, 5, 7, 10},

(a) List A ∪ B.
(b) List A ∩ B.

(a) A ∪ B: join the two sets A ∪ B = {1, 2, 3, 5, 7, 9, 10}

(b) A ∩ B: the elements common is both A ∩ B = {3, 5, 7}

Example 2

Two sets, A and B, is disjoint. If P(A) = 0.4 P(B) = 0.3,

(a) what is the probability of A and B occurring?
(b) what is the probability of A or B occurring?

a) The word refers to P(A∩B):

$P(A \cap B) = 0$

b) The word **or** refers to P(AUB):

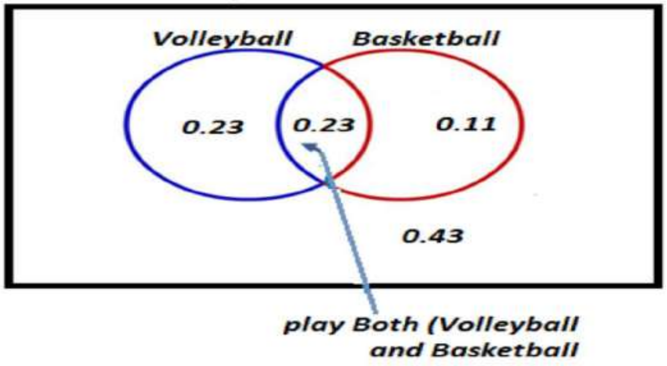
$P(A\,UB) = P(A) + P(B)\text{-}\,P(A \cap B)$
= 0.4 + 0.3 - 0
= 0.7

Example 3

In a school, the probability that a student can play volleyball is 0.46 and the probability that a student can play basketball is 0.34. The probability that the student can play both is 0.23. Find the probability that a randomly selected student can play:

(a) Volleyball only.
(b) volleyball or basketball.
(c) Neither volleyball nor basketball.

Consider a Venn diagram:



*Total Probability to be one so subtract from **one** This is the compliment so 'neither nor'*

(a) Volleyball only = 0.23

(b) volleyball or basketball = Union so add the values of the two circles = 0.57

or Use the formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.46 + 0.34 - 0.23$$
$$= 0.57$$

(c) Neither volleyball nor basketball - outside the two overlapping circles

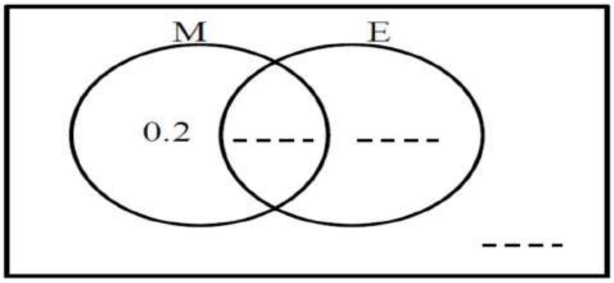
$$\text{Outside} = 1 - (0.23 + 0.23 + 0.11)$$
$$= 1 - 0.57$$
$$= 0.43$$

Example 4

The probability of Tomasi passing Mathematics in Fiji Year 12 Certificate Examination is 0.7, while the probability that he passes English is 0.6. The probability that he passes both subjects is 0.5.

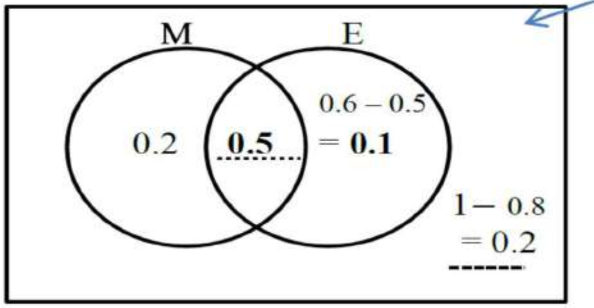
(a) Represent the above probabilities in the Venn diagram given.

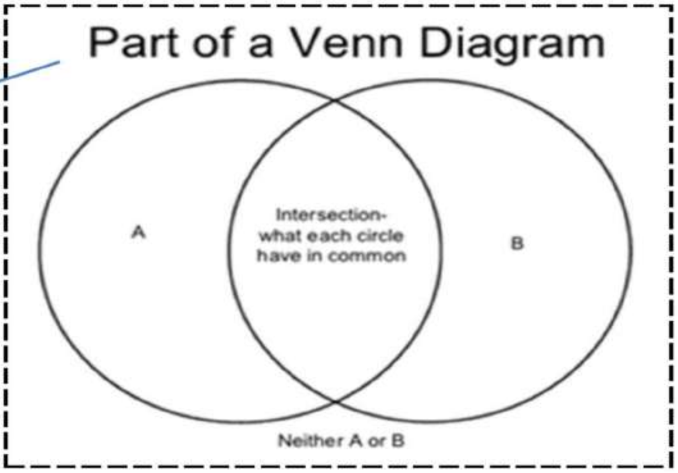
M = probability of passing Mathematics.
E = probability of passing English.



(b) What is the probability that he **fails** both?

Answer:

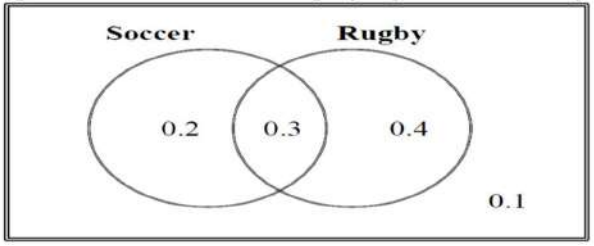




(b) P (failing both) = 0.2

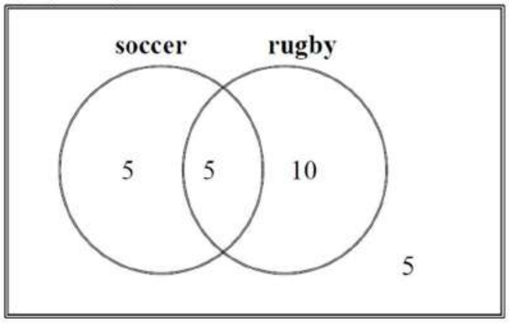
Activity

1. The Venn diagram shows the probabilities of students playing soccer and rugby.



What is the probability that a student plays at least one sport?

2. The Venn diagram shows sports played by a class of Year 12 students.



What is the probability that a student chosen at random from this class plays rugby only?

3. In a school, the probability that a student can play volleyball is 0.46, the probability that the student can play basketball is 0.34 and the probability that the student can play both is 0.23. Find the probability that a randomly selected student can play:
   (a) volleyball only.
   (b) neither volleyball nor basketball.

4. In a small town, the probability that a household has a TV set is 0.73 and the probability that a household has a computer is 0.42. The probability that a household has a TV set and a computer is 0.18. Find the probability that a randomly selected household has:
   (a) a TV set only.
   (b) neither a TV set nor a computer.

Strand 7: Statistics **Sub Strand 7.1: Statistical Analysis**

Lesson 79: Expected Number

Learning Outcome: Calculate expected value.

Recall that probability is the chance that an event will happen. We can use this probability to predict the number of times an event will happen in the future. This in known as Expected value. To find the Expected number, multiply the probability of the outcome by the number of trials.

$$\boxed{E = n \times p}$$

where
E – Expected number
n – Number of trials
p – Probability of event

Example 1

A biased coin, with probability of heads = 0.42, is tossed 300 times. What is the expected number of heads that will be obtained? [Biased is where the outcomes are not equally likely]

$$\begin{aligned} E &= n \times p \\ &= 300 \times 0.42 \\ &= 126 \end{aligned}$$

Example 2

When you toss a fair coin, the probability of getting a tail is 0.5.

(i) Write the probability as a percentage.

(ii) If the coin is tossed 200 times, how many times do you expect to get tails?

(i) $0.5 \times 100 = 50\%$

To convert to percentage,
multiply by 100

Percentage = 50%

(ii) $E = n \times p$

$= 200 \times 0.5$

$= 100$

Expected number = 100

Example 3

A die is to be rolled.

(i) What is the probability that it will show an odd number?

(ii) If it is rolled 180 times, how many times would you expect to get an odd number?

(i) P(odd)=

An **odd number** is an integer which is not a multiple of two.
In a die, the odd numbers are 1, 3, 5

Probability (odd) = $\frac{1}{2}$ or $\frac{3}{6}$ or 0.5 or 50%

(ii) $E = 180 \times \frac{1}{2}$

$= 90$

Using the formula:
$E(x) = n \times p$, where,
$E(x)$ = expected number
n = number of trials
p = probaility

Expected number = 90

Activity

1. An uneven dice with eight equally likely outcomes is tossed. The probability of getting "three" is 0.2.
   (a) Write this probability as a percentage.
   (b) If you toss this dice 30 times, how many times would you expect to get "three"?

2. A math teacher said that the probability of a surprise test on any day this year is 0.15.
   If there are 180 school days in the year, how many surprise tests can you expect to have?

3. A box contains 2 red balls and a yellow ball. The probability of picking a red ball is $\frac{2}{3}$.
   (a) What is the probability of picking a yellow ball?
   (b) If you try to pick 12 times, how many times would you expect it to be yellow?
   (c) How many times would you expect it to be red if you try 36 times?

4. Alisi is a shooter for her school's netball team. In today's game, she has made 12 goals out of 20 attempts.
   (a) What is the probability of the successful shot of the day?
   (b) How many of her next 30 shots, would you expect her to make goals?

RATU NAVULA COLLEGE
YEAR 12 MATHEMATICS - WORKSHEET 8

Strand 8: Probability **Sub Strand 8.1: Probability Experiments**

1. Two dice are rolled and the numbers on the uppermost faces are observed. What is the probability of observing

   (a) number greater than 4 on both dice?
   (b) two odd numbers?

2. Rita has a bag that contained 8 balls of the same size, of which there were 3 red, 3 blue, and 2 white balls. She picked a ball at random, noted its colour, and picked another one **without** replacing the first ball.

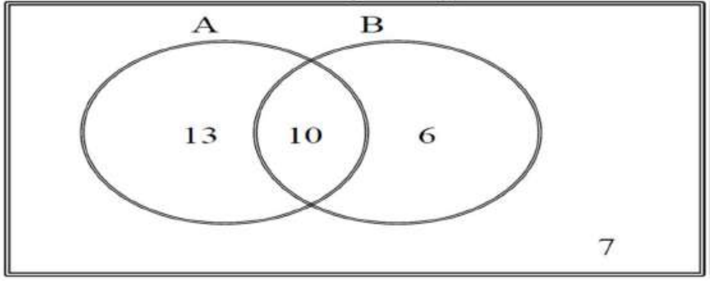
   What is the probability that the balls picked were the same colour?

3. A box contains one green and three yellow balls, all of same size and shape. One ball is withdrawn at random with its colour being noted. **Without** replacing this ball, another is randomly withdrawn and the colour is again noted.

   (a) List the sample space (set of all possible outcomes) for the above experiment (use G for green ball and Y for yellow ball).
   (b) Find the probability of withdrawing a yellow ball on the second draw.

4. The Venn diagram shown below illustrates the results of a survey of the subjects taken by a class of Year 12 students.

   A = number of students taking Agriculture.
   B = number of students taking Biology.



   (a) How many students were surveyed?
   (b) One student is picked at random from this class. What is the probability that the student takes Agriculture?

5. In a local school, the probability that a student has driven a car in the last year is 0.78, the probability that the student has driven a truck is 0.15 and the probability that the student has driven both vehicles is 0.06. Find the probability that a random selected student has

   (a) driven a car only.
   (b) driven one type of vehicle but not the other.
   (c) not driven any type of vehicle in the last year.