# Y11 MATHEMATICS LIFESKILLS HOME LEARNING KIT – WK 6

**STRAND 4 TRIGONOMETRY IN EVERYDAY CONTEXT**

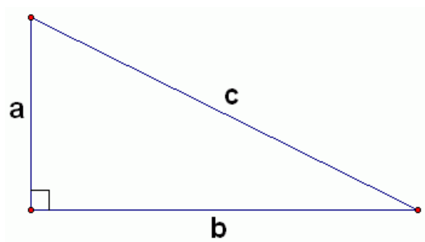
**SUBSTRAND 4.1 APPLICATIONS OF PYTHAGORAS THEOREM**

**LESSON 55**

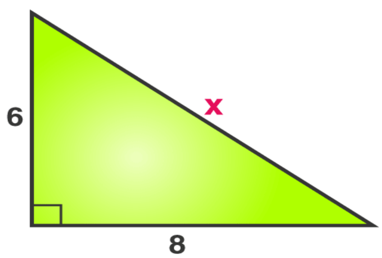
LO: Apply Pyathoras theorem to workout side of a right angled triangle.

Pythagoras theorem states that for all right-angled triangles, **'The square on the hypotenuse is equal to the sum of the squares on the other two sides'**. The hypotenuse is the longest side and it's always opposite the right angle.

$$c^2 = a^2 + b^2$$

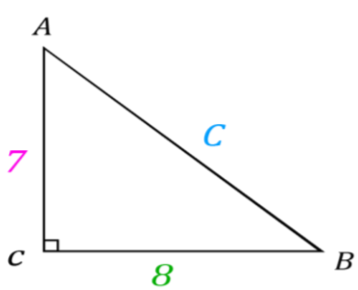



**Example 1**
Consider a right triangle, given below. Find the value of x.



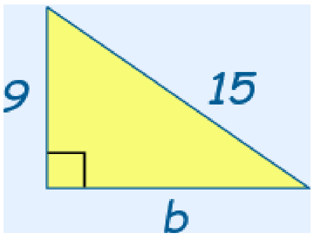

$$c^2 = a^2 + b^2$$
$$x^2 = 6^2 + 8^2$$
$$x^2 = 36 + 64$$
$$x^2 = 100$$
$$x = \sqrt{100}$$
$$x = 10$$

**Example 2**
Consider a right triangle, given below. Find the length of side marked c.




$$c^2 = a^2 + b^2$$
$$c^2 = 7^2 + 8^2$$
$$x^2 = 49 + 64$$
$$x^2 = 113$$
$$x = \sqrt{113}$$

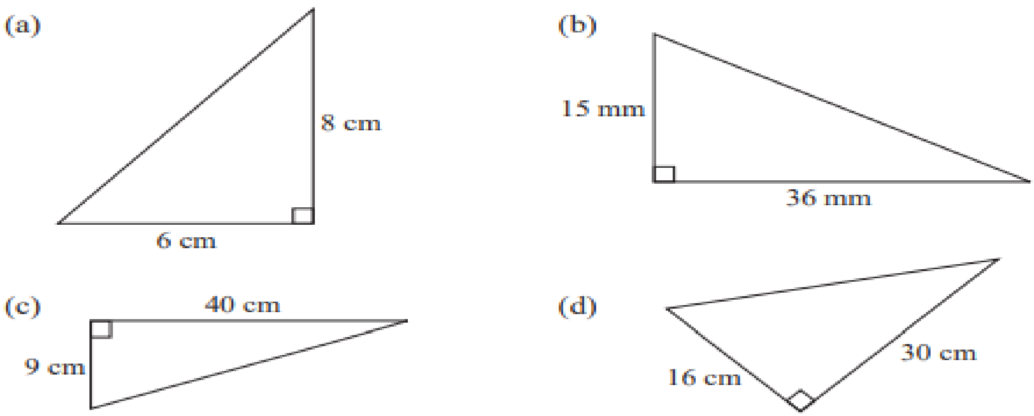
**Example 3**
Find the length of side marked b.




$$c^2 = a^2 + b^2$$
$$15^2 = b^2 + 9^2$$
$$225 = b^2 + 81$$
$$b^2 = 225 - 81$$
$$b^2 = 144$$
$$b = \sqrt{144}$$
$$b = 12$$

**Activity**

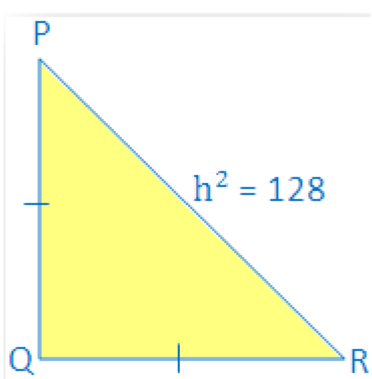
1. Calculate the length of the hypotenuse of each of these triangles.




**Lesson 56**

LO: Solve word problems on Pythagoras Theorem.

**Example 1**
If the square of the hypotenuse of an isosceles right triangle is 128 $cm^2$, find the length of each side.



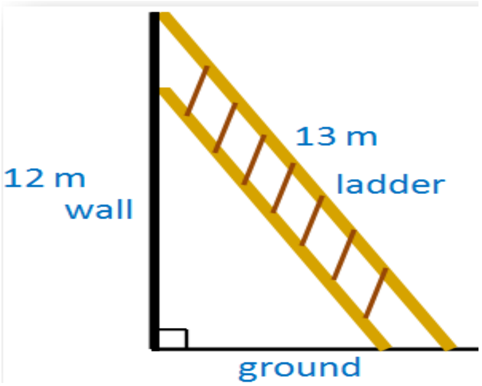

Let the two equal side of right angled isosceles triangle, right angled at Q be *x* cm.

$$c^2 = a^2 + b^2$$
$$PR^2 = PQ^2 + QR^2$$
$$h^2 = x^2 + x^2$$
$$128 = 2x^2$$
$$x^2 = 64$$
$$x = \sqrt{64}$$
$$x = 8$$

Therefore, length of each side is 8 cm.

**Example 2**

A ladder 13 m long is placed on the ground in such a way that it touches the top of a vertical wall 12 m high. Find the distance of the foot of the ladder from the bottom of the wall.




$$c^2 = a^2 + b^2$$
$$13^2 = x + 12^2$$
$$169 = b^2 + 144$$
$$x^2 = 169 - 144$$
$$b^2 = 25$$
$$b = \sqrt{25}$$
$$b = 5\ m$$

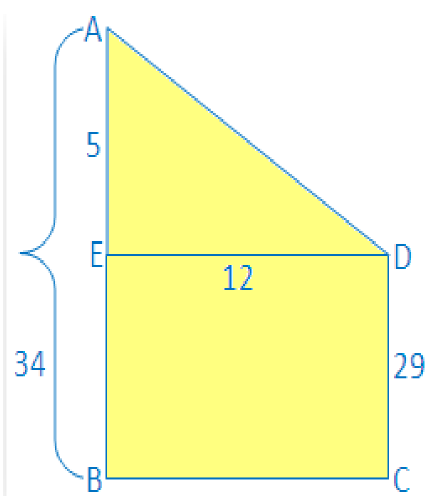
**Activity**

1. Jonah is deep-sea diving with two friends. Akira is exploring a coral reef 16m in front of Jonah, and Valeria is floating on the surface directly above Jonah. If Valeria and Akira are 20m apart, how far apart are Jonah and Valeria?

2. Three people are sitting in a bus. Sandeep is seated 4m directly behind Hanson and 3m directly left of Amy. How far is Hanson from Amy?

**Lesson 57** LO: Solve practical situations using Pythagoras Theorem.

**Example 1**
The height of two building is 34 m and 29 m respectively. If the distance between the two building is 12 m, find the distance between their tops.

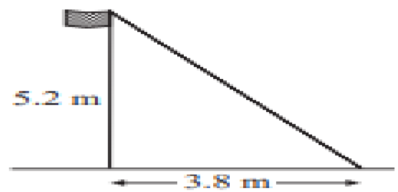



The vertical buildings AB and CD are 34 m and 29 m respectively.
Draw DE ⊥ AB
Then AE = AB – EB but EB = BC
Therefore AE = 34 m - 29 m = 5 m
Now, AED is right angled triangle and right angled at E.

$$c^2 = a^2 + b^2$$
$$AD^2 = 5^2 + 12^2$$
$$AD^2 = 25 + 144$$
$$AD^2 = 169$$
$$AD = \sqrt{169}$$
$$AD = 13\ \mathrm{m}$$

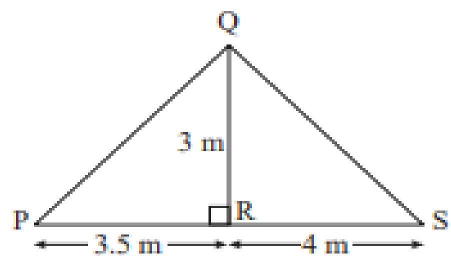
## Activity

1. One end of a rope is tied to the top a vertical flagpole of height 5.2 m. When the rope is pulled tight, the other end is on the ground 3.8 m from the base of the flagpole. Calculate the length of the rope.



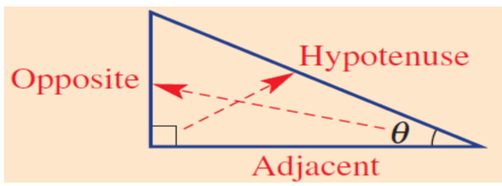

2. The diagram shows a wooden frame that is to be part of the roof of house.

   a) Use Pythagoras' Theorem in triangle PQR to find the length of PQ.
   b) Calculate the length of QS.
   c) Calculate the total length of wood needed to make the frame.

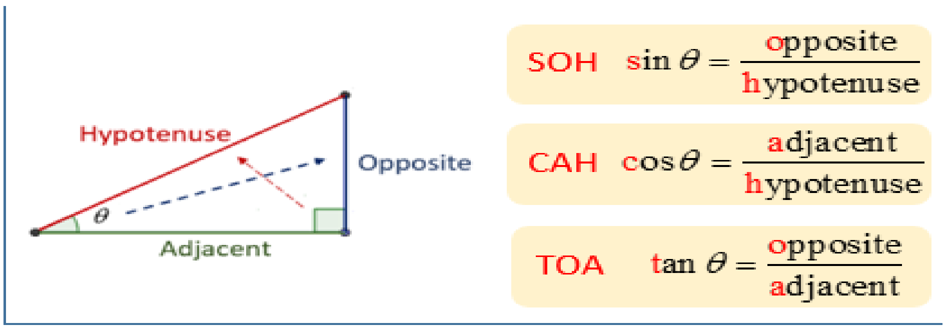



**Lesson 58** LO: Determine length or angle using Trigonometric ratios.

For a right-angled triangle with another angle named :

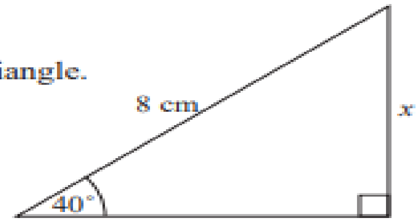



- The hypotenuse is the longest side, opposite the 90° angle
- The opposite side is opposite $\theta$
- The adjacent side is next to $\theta$ but not the hypotenuse.



## Example 1

Calculate the length of the side marked $x$ in this triangle.




## Solution

In this question we use the *opposite* side and the *hypotenuse*. These two sides appear in the formula for $\sin\theta$, so we begin with,

$$\sin\theta = \frac{O}{H}$$

In this case this gives,

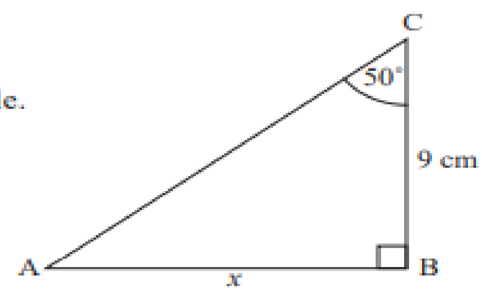
$$\sin 40° = \frac{x}{8}$$

or

$$\begin{aligned} x &= 8 \times \sin 40° \\ &= 5.142300877 \text{ cm} \\ &= 5.1 \text{ cm to 1 decimal place} \end{aligned}$$

## Example 2

Calculate the length of the side AB of this triangle.




## Solution

In this case, we are concerned with side A B which is the *opposite* side and side BC which is the *adjacent* side, so we use the formula,

$$\tan\theta = \frac{O}{A}$$

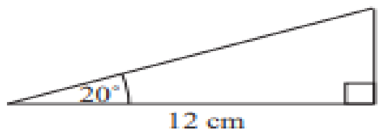
For this problem we have,

$$\tan 50° = \frac{x}{9}$$

$$\begin{aligned} \text{so} \quad x &= 9 \times \tan 50° \\ &= 10.72578233 \text{ cm} \\ &= 10.7 \text{ cm to 1 decimal place} \end{aligned}$$

## Example 3

Calculate the length of the hypotenuse of this triangle.




### Solution

In this case, we require the formula that links the *adjacent* side and the *hypotenuse*, so we use $\cos\theta$.

Starting with

$$\cos\theta = \frac{O}{H}$$

we can use the values from the triangle to obtain,

$$\cos 20° = \frac{12}{H}$$

$$H \times \cos 20° = 12$$

$$H = \frac{12}{\cos 20°}$$

$$= 12.77013327 \text{ cm}$$

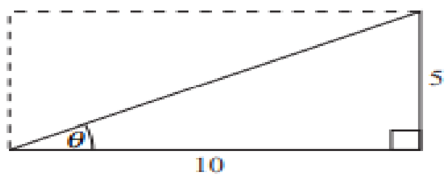
Therefore the hypotenuse has length 12.8 cm to 1 decimal place.

## Example 4

A rectangle has sides of length 5 m and 10 m. Determine the angle between the long side of the rectangle and a diagonal.

### Solution

The solution is illustrated in the diagram.




Using the formula for $\tan\theta$ gives

$$\tan\theta = \frac{5}{10}$$

$$= 0.5$$

Then using the $\tan^{-1}$ key on a calculator gives

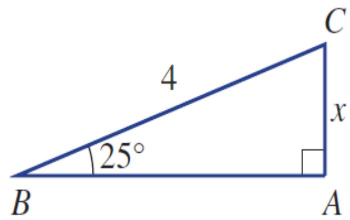
$$\theta = \tan^{-1}(0.5) = 26.56505118°$$

$$= 26.6° \quad \text{(to 1 decimal place).}$$

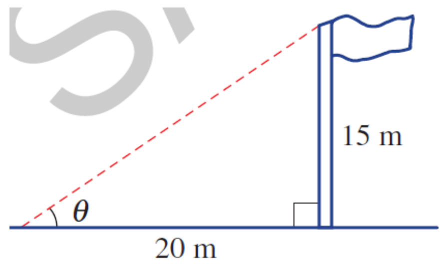
**Activity**

1. For this right-angled triangle:

   (a) Find the value of length AB.
   (b) Calculate the value of x correct to three decimal places using the sine ratio.
   (c) Calculate the value of x correct to three decimal places but instead use the cosine ratio.

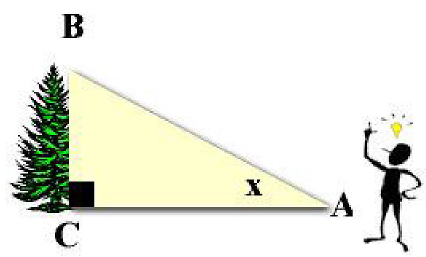



2. A vertical flagpole casts a shadow 20 m long. If the flagpole is 15m high, find the value of $\theta$.

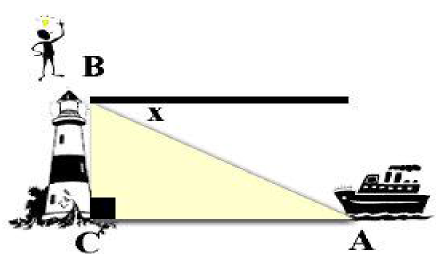



**Lesson 59** LO: Solve practical situations using Trigonometric ratios.

The angle of elevation is always measured from the ground up. It is always INSIDE the triangle. In the diagram, x marks the angle of elevation of the top of the tree as seen from a point on the ground. You can think of the angle of elevation in relation to the movement of your eyes. You are looking straight ahead and you must raise (elevate) your eyes to see the top of the tree.

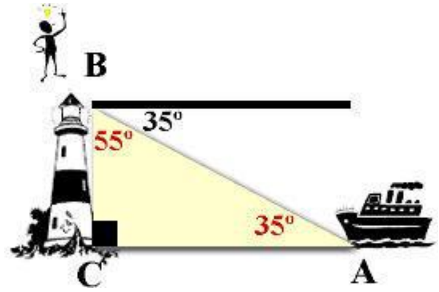



The angle of depression is always OUTSIDE the triangle. In the diagram at, x marks the angle of depression of a boat at sea from the top of a lighthouse. You can think of the angle of depression in relation to the movement of your eyes. You are standing at the top of the lighthouse and you are looking straight ahead. You must lower (depress) your eyes to see the boat in the water.



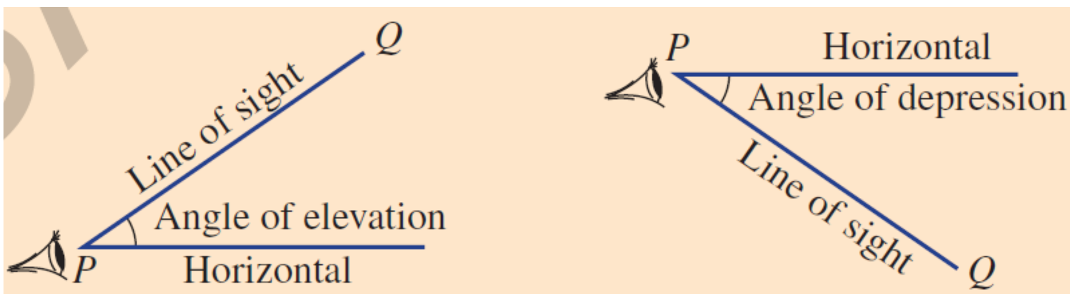
**EXAMPLE**

There are two possible ways to use our angle of depression to obtain an angle INSIDE the triangle.



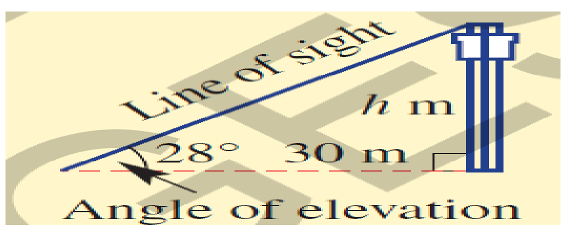

1. Find the angle adjacent (next door) to our angle. This adjacent angle will always be the complement of our angle. Our angle and the angle next door will add to 90°. In the diagram on the left, the adjacent angle is 55°.

2. Utilize the fact that the **angle of depression = the angle of elevation** and simply place 35° in angle A. (the easiest method)




**Activity**

1. The angle of elevation of the top of a tower from a point on the ground 30 m away from the base of the tower is 28°. Find the height of the tower to the nearest metre.




2. A plane flying at 1850 m starts to climb at an angle of 18° to the horizontal when the pilot sees a mountain peak 2450 m high, 2600 m away from him in a horizontal direction. Will the pilot clear the mountain?