RATU NAVULA COLLEGE
Y11 MATHEMATICS LIFESKILLS SUPPLEMENTARY NOTES 4

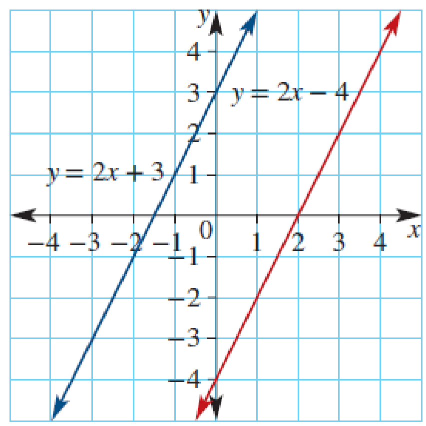
Strand 3.0 **Linear Functions in Everyday Context**

Sub-Strand 3.2 **Modelling using Simultaneous Linear Equations**

Lesson 45 LO: Compare parallel lines.

Parallel lines

Consider the linear function $y = 2x + 3$.
It has a gradient of 2 and y-intercept of 3.
Consider the linear function $y = 2x - 4$.
It has a gradient of 2 and y-intercept of -4.
The graph of these linear functions is shown opposite.
They are parallel because they both have the same gradient of $m = 2$.



If the value of *m* is the same for two linear functions, then the lines are parallel.

<u>ACTIVITY</u>

1.

Which of the following lines are parallel?

a $y = 2x + 1$ and $y = x + 2$

b $y = -2x + 4$ and $y = 2x + 4$

c $y = 3x + 1$ and $y = 3x + 2$

d $y = -4x + 1$ and $y = -4x$

LESSON 46 LO: Solve Simultaneous equations – graphically

Simultaneous Equations

Simultaneous Equations are equations that apply at the same time. Here we consider a pair of equations in two unknowns, x and y.

The solution will be the numbers x and y that satisfy both equations.

Methods of Solving Simultaneous Equations

- Substitution Method
- Elimination Method

Two straight lines will always intersect unless they are parallel.

SOLVING A PAIR OF SIMULTANEOUS EQUATIONS GRAPHICALLY

1. Draw a number plane.
2. Graph both linear equations on the number plane.
3. Read the point of intersection of the two straight lines.
4. Interpret the point of intersection for practical applications.

Example Finding the solution of simultaneous linear equations

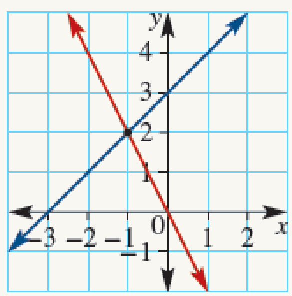
Find the simultaneous solution of $y = x + 3$ and $y = -2x$.

SOLUTION:

1 Use the gradient–intercept form to determine the gradient (coefficient of x) and y-intercept (constant term) for each line.

$y = x + 3$
Gradient is +1, y-intercept is 3.
$y = -2x$
Gradient is −2, y-intercept is 0.

2 Draw a number plane.
3 Sketch $y = x + 3$ using the y-intercept of 3 and a gradient of 1.
4 Sketch $y = -2x$ using the y-intercept of 0 and a gradient of −2.



5 Find the point of intersection of the two lines.

(−1, 2)

6 The simultaneous solution is the point of intersection.

Simultaneous solution is $x = -1$ and $y = 2$, (−1, 2).

7 Alternatively, construct a table of values for x and y. Let $x = -2, -1, 0, 1$ and 2. Find y using the linear function $y = x + 3$.

x	−2	−1	0	1	2
y	1	2	3	4	5

8 Repeat to find y using the linear function $y = -2x$.

x	−2	−1	0	1	2
y	4	2	0	−2	−4

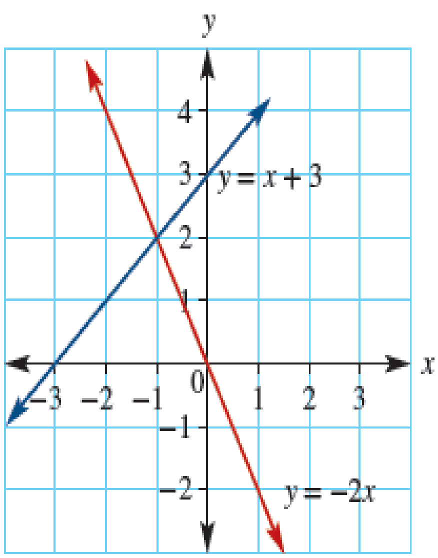
9 The same value of x and y occurs in both tables when $x = -1$ and $y = 2$.

Simultaneous solution is $x = -1$ and $y = 2$.

ACTIVITY

1. What is the point of intersection for each of these pairs of straight lines?

a



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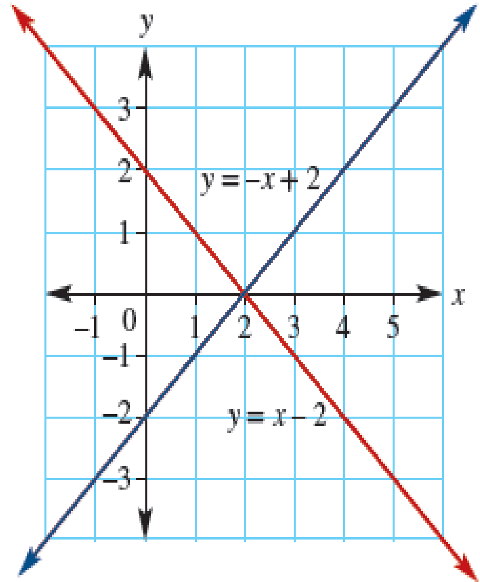

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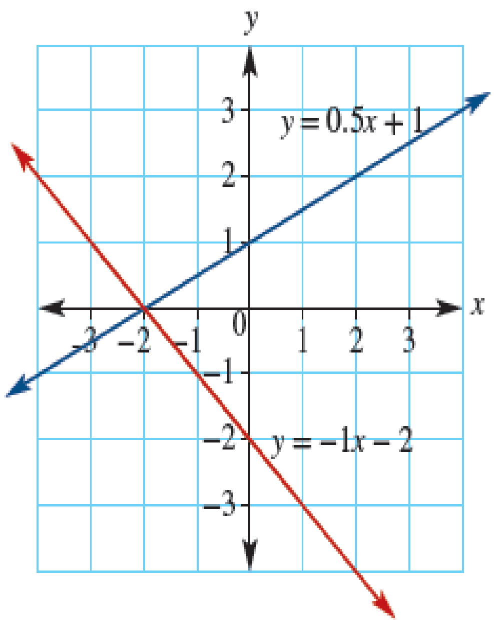
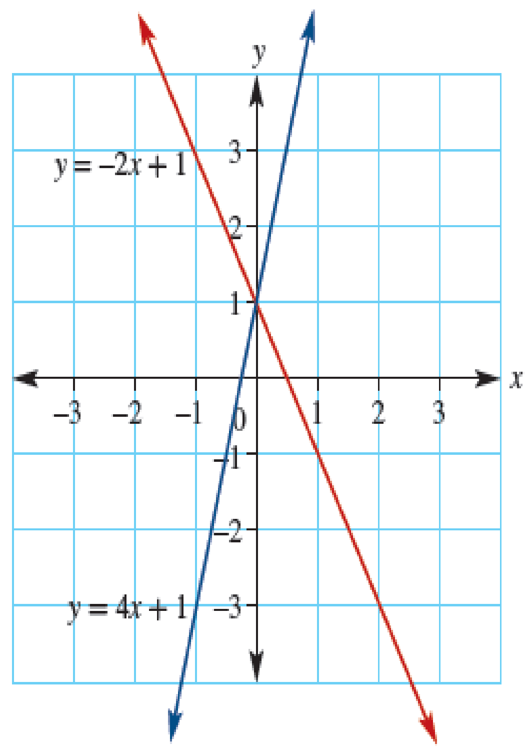


d

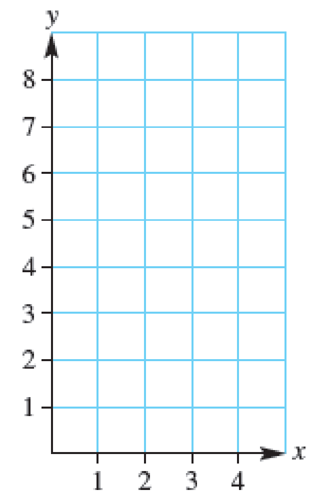


2.

Plot the following points on a number plane and join them to form two straight lines. What is the point of intersection of these straight lines?

x	0	1	2	3	4
y	0	2	4	6	8

x	0	1	2	3	4
y	6	5	4	3	2



LESSON 47 LO: Solve simultaneous equations- algebraically

A linear equation such as $y = 2x + 3$ has two variables, x and y. It has many solutions; for example, $x = 0$ and $y = 3$ or $x = 1$ and $y = 5$. However, when there are two linear equations there is only one solution unless the two lines are parallel. The two equations are called simultaneous equations. The solution of the simultaneous equations represents the point of intersection of the two lines.

The elimination method and the substitution method will result in the same solution. If the simultaneous equations have either x or y as the subject it is often easier to use the substitution method.

Method 1: Substitution

When solving simultaneous equations by substitution, the process is to substitute a variable from one equation into the other equation.

SIMULTANEOUS EQUATIONS: SUBSTITUTION METHOD

1. Make one pronumeral the subject in one of the equations.
2. Substitute the expression for this subject into the other equation.
3. Solve this new equation to find the value of one pronumeral.
4. Substitute this value into one of the equations to find the value of the second pronumeral.

Example 1 : Solving simultaneous equations by substitution

Solve this pair of simultaneous equations: $y = 2x + 3$ and $y = -x$.

SOLUTION:

1	Number the two equations as (1) and (2).	$y = 2x + 3$ $y = -x$	(1) (2)
2	Substitute the y-value from equation (2), which is $-x$, into equation (1).	$-x = 2x + 3$	
3	Solve the equation for x by subtracting $2x$ from both sides of the equation.	$-3x = 3$	
4	Divide both sides of the equation by -3.	$x = -1$	
5	To find y, substitute $x = -1$ into equation (2).	$y = -(-1) = 1$	
6	Check the solution by substituting $x = -1$ and $y = 1$ into equation (1).	Check: $y = 2x + 3$ $1 = 2 \times (-1) + 3$ True	
7	Write the answer in words.	Solution is $x = -1$ and $y = 1$, $(-1, 1)$.	

Substitution Method

This is a preferred method when one equation has one of the variables as its subject.

STEPS

1. Solve one of the equations for one unknown in terms of the other.
2. Then, substitute that in the other equation.
3. That will yield one equation in one unknown, which we can then solve.

Example 2

Solve these two equations simultaneously

$2x + y = 4$ and $x - y = -1$

Solve the first equation for y

$2x + y = 4 \qquad y = 4 - 2x$

Substitute the y in the second equation

$$\begin{aligned} x - y &= -1 \\ x - (4 - 2x) &= -1 \\ x - 4 + 2x &= -1 \\ 3x - 4 &= -1 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

To find y

$$\begin{aligned} y &= 4 - 2x \\ y &= 4 - 2(1) \\ y &= 2 \end{aligned}$$

EXAMPLE 3

Example: Solve simultaneously

$$y = 3x - 1$$

$$-2x + 7y = 31$$

Label the equations

$$y = 3x - 1 \text{ ---- } (1)$$

$$-2x + 7y = 31 \text{ ---- } (2)$$

Substitute (1) into (2) and solve for x

$$-2x + 7(3x - 1) = 31$$

$$-2x + 21x - 7 = 31$$

$$19x - 7 = 31$$

$$19x = 31 + 7$$

$$19x = 38$$

$$x = \frac{38}{19}$$

$$x = 2$$

Substitute x=2 into (1)

$$y = 3x - 1$$

$$y = 3 \times 2 - 1$$

$$y = 5$$

The solution is x = 2 and y=5

ACTIVITY

1. 1000 tickets were sold. Adult tickets cost \$8.50, children's cost \$4.50, and a total of \$5700 was collected. How many tickets of each kind were sold?

2. Samantha has 30 coins, consisting of 20c and 50c, which total \$13.50. How many of each does she have?

LESSON 48 LO: Solve equations using Elimination Method

Method 2: Elimination

This works by ‘eliminating’ or removing one of the variables to get a single equation in one variable. This is done by either adding or subtracting the two equations. To eliminate a pronumeral requires the coefficients of one pronumeral to be exactly the same or the same but opposite in sign.

STEPS

1. Make sure that the two coefficients of one pronumeral are the same. This may require multiplying or dividing one or both equations by a number.
2. Eliminate one pronumeral by adding or subtracting the two equations.
3. Solve this new equation to find the value of one pronumeral.
4. Substitute this value into one of the equations to find the value of the second pronumeral.

Example 1: Solving simultaneous equations by elimination

Solve the pair of simultaneous equations $y = 2x + 3$ and $y = -x$.

SOLUTION:

1	Number the two equations as (1) and (2).	$y = 2x + 3$ $y = -x$	(1) (2)
2	Subtract equation (2) from equation (1).	$y - y = 2x + 3 - (-x)$	
3	To solve the equation for x, eliminate y (since $y - y = 0$) and add the like terms ($2x - (-x) = 3x$).	$0 = 3x + 3$	
4	Subtract $-3x$ from both sides of the equation.	$-3x = 3$	
5	Divide both sides of the equation by $-3x$ to give x.	$x = -1$	
6	To find y, substitute $x = -1$ into equation (2).	$y = -(-1) = 1$	
7	Check the solution by substituting $x = -1$ and $y = 1$ into equation (1).	Check: $y = 2x + 3$ $1 = 2 \times (-1) + 3$ True	
8	Write the answer in words.	Solution is $x = -1$ and $y = 1$, $(-1, 1)$.	

Elimination Method

In the 'elimination' method for solving simultaneous equations, two equations are simplified by adding them or subtracting them.

Example 2

Solve these two equations simultaneously

$2x - 5y = 1$ and $3x + 5y = 14$

The first equation contains a '-5y' term, while the second equation contains a '+5y' term. These two terms will cancel if added together, so we will add the equations to eliminate 'y'.

$$\begin{array}{rl} & 2x - 5y = 1 \\ + & \underline{3x + 5y = 14} \\ & 5x = 15 \\ & x = 3 \end{array}$$

By substituting 3 for 'x' into either of the two original equations we can find 'y'.

$2x - 5y = 1$

$2(3) - 5y = 1$

$6 - 5y = 1$

$-5y = -5$

$y = 1$

Example 3
Solve these two equations simultaneously
$4x - 3y = 3$ and $10x + 3y = 4$

$$\begin{array}{rrcl} & 4x - 3y & = & 3 \\ + & 10x + 3y & = & 4 \\ \hline & 14x & = & 7 \\ & x & = & \frac{1}{2} \end{array}$$

$$\begin{aligned} 4x - 3y &= 3 \\ 4\left(\frac{1}{2}\right) - 3y &= 3 \\ 2 - 3y &= 3 \\ y &= -\frac{1}{3} \end{aligned}$$

Example 4
Solve these two equations simultaneously
$2x + 3y = 4$ and $x - 2y = -5$

Before using the elimination method you may have to multiply every term of one or both of the equations by some number so that equal terms can be eliminated.
By multiplying every term in the second equation by 2 :

$$2 \times (x - 2y = -5) \qquad 2x - 4y = -10$$

Now the 'x' term in each equation is the same, and the equations can be subtracted to eliminate 'x':

$$\begin{array}{rrcl} & 2x + 3y & = & 4 \\ - & 2x - 4y & = & -10 \\ \hline & 7y & = & 14 \\ & y & = & 2 \end{array}$$

The other variable, 'x', can now be found by substituting 2 for 'y' into either of the original equations.

$$\begin{aligned} x - 2y &= -5 \\ x - 2(2) &= -5 \\ x - 4 &= -5 \\ x &= -1 \end{aligned}$$

ACTIVITY

1. 7 cups of coffee and 4 pieces of toast cost $20. 5 cups of coffee and 3 pieces of toast cost $14.50. Find the cost of each item

2. 3 books and 2 pens cost $8. 5 books and 3 pens cost $13. Find the total cost of one book and one pen.

LESSON 49 LO: Model using Simultaneous equation.

Simultaneous equation models

When two practical situations are described mathematically using a linear function then the point of intersection has an important and often different meaning depending on the situation. For example, when income is graphed against costs the point of intersection represents the point where a business changes from a loss to a profit.

SIMULTANEOUS EQUATIONS AS MODELS

Simultaneous equation models use two linear functions to describe a practical situation and the point of intersection is often the solution to a problem.

Example 1: Using simultaneous equations as models

Zaina buys and sells books. Income received by selling a book is calculated using the formula $I = 16n$. Costs associated in selling a book are calculated using the formula $C = 8n + 24$.

a. What is the income when 6 books are sold?
b. What is the costs when 6 books are sold?
c. Draw the graph of $I = 16n$ and $C = 8n + 24$ on same number plane.
d. Use the graph to determine the number of books needed to be sold for the costs to equal the income.

SOLUTION

1. Substitute 6 for n into the formula for income $I = 16n$.

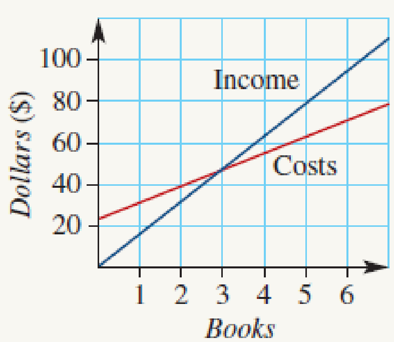
a $I = 16n = 16 \times 6 = 96$
$\therefore$ Income for six books is \$96

2. Substitute 6 for n into the formula for costs $C = 8n + 24$.

b $C = 8n + 24 = 8 \times 6 + 24 = \$72$
$\therefore$ Costs for six books is \$72

3. Draw a number plane.
4. Use the gradient–intercept form to determine the gradient and vertical intercept for each line. Gradient is the coefficient of n. Vertical intercept is the constant term.
5. Sketch $I = 16n$ using the vertical intercept of 0 and gradient of 16.
6. Sketch $C = 8n + 24$ using the vertical intercept of 24 and gradient of 8.

c



7. Find the point of intersection of the two lines (3, 48).

d Income is equal to costs when $n = 3$
$\therefore$ 3 books

Example 2

A total of 925 tickets were sold for $5925. If adult tickets cost $7.50 and children's tickets cost $3.00, how many tickets of each kind were sold?

Let x be the number of adult tickets.
Let y be the number of children's tickets.

$$x + y = 925$$
$$7.5x + 3y = 5925$$

$$\begin{array}{rl} 4x - 3y &= 2775 \\ -\ \underline{7.5x + 3y} &\underline{= 5925} \\ -4.5x &= -3150 \\ x &= 700 \end{array}$$

$$\begin{array}{rl} x + y &= 925 \\ 700 + y &= 925 \\ y &= 225 \end{array}$$

700 adult tickets and 225 children's tickets.

EXAMPLE 3

Madhu bought 3 packets of peanuts and 2 packets of bongo for $4. Rina bought 3 packets of peanuts and 4 packets of bongo for $5

Let the cost of a packet of peanut be x and a packet of bongo be y.

(a) Write an equation for Madhu's purchases?

3 packets of peanuts + 2 packets of bongo = $4
3x + 2y = 4

(b) Write an equation for Rina's purchases?

3 packets of peanuts and 4 packets of bongo for $5
3x + 4y =5

(c) Solve the two equations simultaneously and find the cost of peanuts and bongos.

$$3x + 2y = 4 \text{ ---- } (1)$$
$$3x + 4y = 5 \text{ ---- } (2)$$

Eliminate x by subtracting the two equations

$$\begin{array}{r} 3x+2y=4 \\ -\ 3x+4y=5 \\ \hline \end{array}$$

$$-2y=-1$$

$$y=\frac{-1}{-2}$$

$$y=0.5$$

Substitute y= 0.5 into (1) to find x

$$3x+2y=4$$

$$3x+2\times 0.5=4$$

$$3x+1=4$$
$$3x=4-1$$

$$3x=3$$

$$x=\frac{3}{3}=1$$

Hence a packet of peanut costs $1 and a packet of bongo costs $0.50

ACTIVITY

1. If the cost of 3 chocolates and 2 cookies is $22 and that of 2 chocolates and 3 cookies is $18, what is the cost of cookies?

2. 2 tables and 3 chairs together cost $1900 whereas 3 tables and 2 chairs together cost $2600 . Find the cost of a table and a chair.