Ratu Navula College

Year 11 Applied Mathematics Lesson Notes – Week 3

Lesson 36

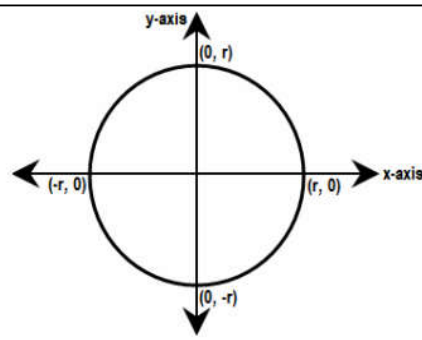
Strand 4: Graphs
Sub- Strand 4.1 : Graphs
Learning Outcome : sketch a circular graph.

GRAPH OF CIRCLE CENTERED AT THE ORIGIN

- The set of all points on a plane that are fixed distance from the center.

General Form: $\mathbf{x^2 + y^2 = r^2}$

Centre/Origin: $\{\mathbf{0}, \mathbf{0}\}$

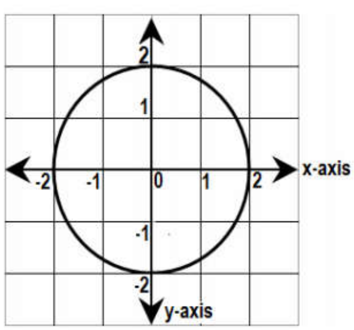


Example 1

Find the equation of the circle with centre (0,0) and radius 2.

$x^2 + y^2 = r^2 \quad (0, 0) \rightarrow center \ \ of \ \ the \ \ circle \quad r = 2 \rightarrow radius \ \ of \ \ the \ \ circle$

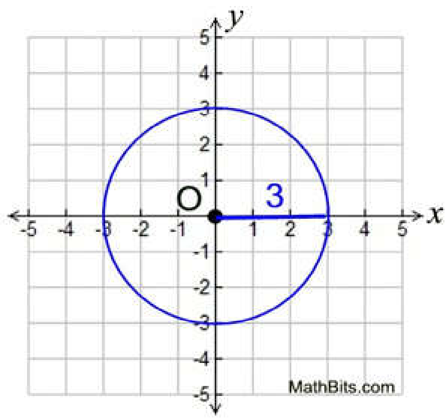
$x^2 + y^2 = r^2 \rightarrow x^2 + y^2 = 2^2 \rightarrow \underline{\underline{x^2 + y^2 = 4}}$



Example 2

Sketch the the graph $x^2 + y^2 = 9$

Solution : the radius is $\sqrt{9} = 3$



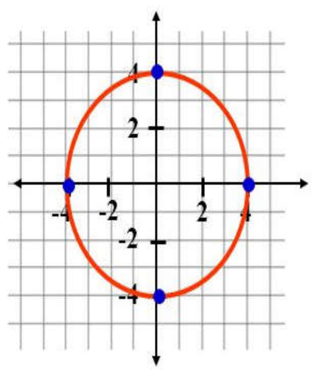
Example 3

Write the standard equation of the circel whose centre is at the origin and whose radius is 4.
Sketch the graph.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 4^2$$

$$x^2 + y^2 = 16$$



Exercise

Sketch the following graph:

(a) $x^2 + y^2 = 36$
(b) $x^2 + y^2 = 25$

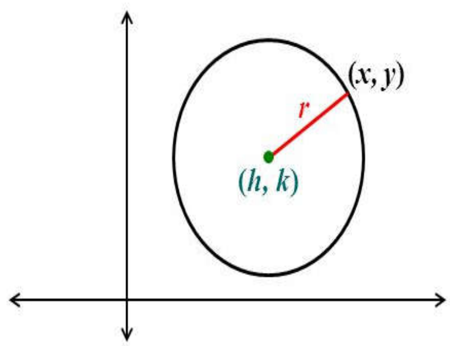
Lesson 37

Strand 4: Graphs
Sub- Strand 4.1: Graphs
Learning Outcome : Determine the radius and sketch the circular graph.

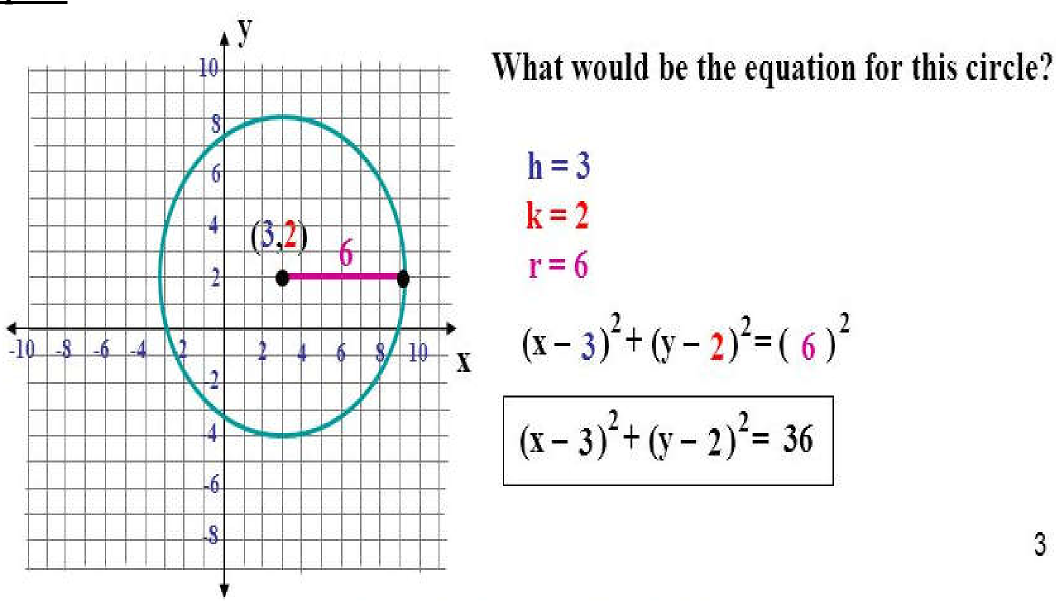
Equation of a circle
The equation of a circle with **centre at** $(\boldsymbol{h}, \boldsymbol{k})$ and the radius r units is :

$$(\boldsymbol{x} - \boldsymbol{h})^{2} + (\boldsymbol{y} - \boldsymbol{k})^{2} = \boldsymbol{r}^{2}$$

Notes : h and k are arbitrary variables.



Example 1



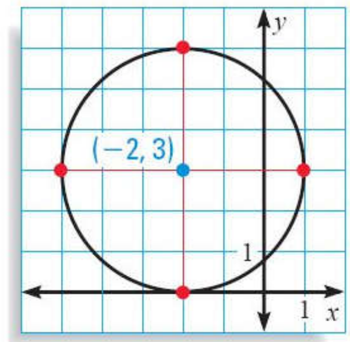
Example 2

The equation of a circle is $(x + 2)^2 + (y - 3)^2 = 9$. Graph the circle.

Rewrite the equation to find the center and radius:

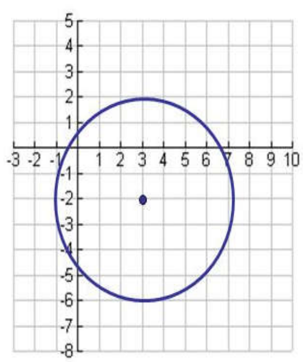
$$(x + 2)^2 + (y - 3)^2 = 9$$

$$[x - (-2)]^2 + (y - 3)^2 = 3^2$$



Example 3

Write the standard form for the equation of a circle with center (3,-2) and radius of 4. Also the sketch the graph.



$$(x-h)^2+(y-k)^2=r^2$$

$$(x-3)^2+(y-(-2))^2=4^2$$

$$(x-3)^2+(y+2)^2=16$$

Exercise

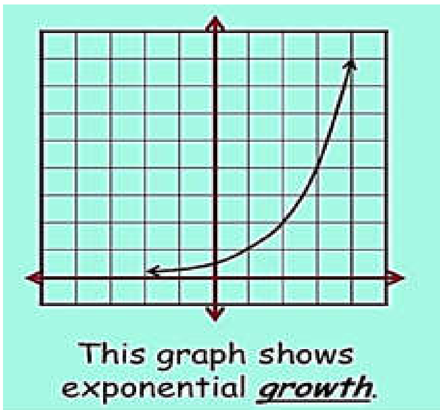
Sketch the following graph:

(a) $(x - 2)^2 + (y - 3)^2 = 4^2$
(b) $(x + 1)^2 + (y - 2)^2 = 16$
(c) $(x)^2 + (y + 2)^2 = 9$

Lesson 38

Strand 4: Graphs
Sub- Strand 4.1: Graphs
Learning Outcome : Sketch an exponetial graph
EXPONENTIAL GRAPH

- General form: $\boldsymbol{y = a^x}$**, where** $\boldsymbol{a} > 0$**.**



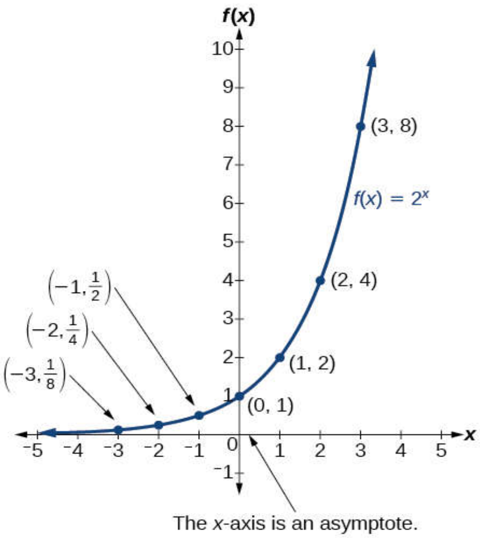
Note:

1. **y-intercept** is always $at\ 1\ (0, 1)$
2. **x-axis** is an **asymptote** (line that the graph gets very close to but never crosses): $y = 0$

Example 1

Graph $y = 2^x$
Solution

x	-3	-2	-1	0	1	2	3
y	1/8	¼	½	1	2	4	8



<u>Example 2</u>

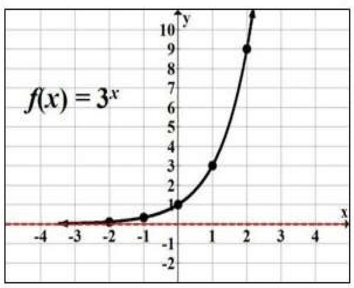
Sketch the graphs of $\boldsymbol{f(x) = 3^x}$ $\quad$ ***and*** $\quad$ $\boldsymbol{f(x) = \left(\frac{1}{3}\right)^x}$

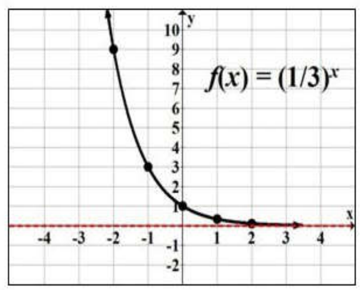
f(x) = 3^x

x	$y = 3^x$
–2	$3^{-2} = 1/9$
–1	$3^{-1} = 1/3$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$

f(x) = $(1/3)^x$

x	$y = (1/3)^x$
–2	$(1/3)^{-2} = 9$
–1	$(1/3)^{-1} = 3$
0	$(1/3)^0 = 1$
1	$(1/3)^1 = 1/3$
2	$(1/3)^2 = 1/9$





<u>Exercise</u>

Sketch the following graph:

(a) $\quad y = 4^x$

(b) $\quad y = \left(\frac{1}{4}\right)^x$

(c) $\quad y = \left(\frac{1}{2}\right)^x$

Lesson 39 – Coordinate Geometry

Strand 5: Coordinate Geometry
Sub Strand: Application of coordinate geometry
Learning Outcome: Calculate the distance between two points using the distance formula.

What are coordinates?

- The coordinates are always written in a certain order that is:
  - Horizontal distance first (x- values)
  - Then **vertical** distance (y- values)
- This is known as **Ordered Pairs**
- Point of origin is known as **(0,0)**

For Example

(2, 3) - This simply means 2 units to the right and 3 units up
(0, -5) - This simply means 0 units to the right and 5 units to the left

Distance Formula

- This formula is used to find the distance between two points only.

$$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Steps:

1. Use the distance formula
2. Label the two points as $\boldsymbol{x_2, x_1, y_2, y_1}$
3. Put the values into the formula
4. Use the order of operation and simplify.

Example 1

Find the length of the line segment whose end points are (5,4) and (-3,4).

$$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$
$$\boldsymbol{= \sqrt{(-3 - 5)^2 + (4 - 4)^2}}$$
$$\boldsymbol{= \sqrt{(-8)}^2}$$
$$\boldsymbol{= \sqrt{64}}$$
$$\boldsymbol{= 8\ units}$$

Example 2

Find the distance between the points P $(-1, 5)$ and Q $(3, -2)$.

THINK	WRITE
1 Let P have coordinates (x_1, y_1).	Let $(x_1, y_1) = (-1, 5)$
2 Let Q have coordinates (x_2, y_2).	Let $(x_2, y_2) = (3, -2)$
3 Find the length PQ by applying the formula for the distance between two points.	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{[3 - (-1)]^2 + (-2 - 5)^2}$ $= \sqrt{(4)^2 + (-7)^2}$ $= \sqrt{16 + 49}$ $= \sqrt{65}$ $= 8.06$ (correct to 2 decimal places)

Class Activity

1. Find the distance between the following points.
   i. (2,5) and (6,8)
   ii. (-1,3) and (-7,-5)

Lesson 40

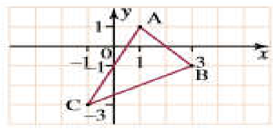
Strand 5: Coordinate Geometry
Sub Strand: Application of coordinate geometry
Learning Outcome: Calculate the distance between more than three points using the distance formula.

How to determine the distance if three or more points are given

1. Label the points using A,B and C
2. Find the 3 lengths (AB,AC,BC) between the points using the distance formula
   Note: if the length cannot be squared rooted evenly, leave in radical form.
3. If it is a right angled triangle then apply Pythagoras theorem $(a^2 + b^2 = c^2)$ to prove if it's a right angled triangle or not

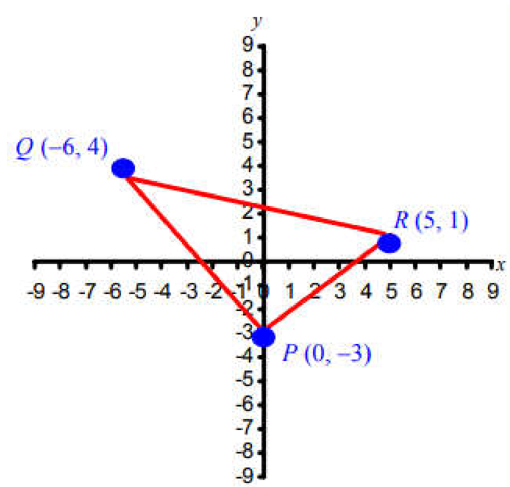
Example 1

Prove that the points A (1, 1), B (3, −1) and C (−1, −3) are the vertices of an isosceles triangle.

THINK	WRITE/DRAW
1 Plot the points and draw the triangle. *Note*: For triangle ABC to be isosceles, two sides must have the same magnitude.	
2 AC and BC seem to be equal. Find the length AC. A $(1, 1) = (x_2, y_2)$ C $(-1, -3) = (x_1, y_1)$	$AC = \sqrt{[1-(-1)]^2 + [1-(-3)]^2} = \sqrt{(2)^2 + (4)^2} = \sqrt{20} = 2\sqrt{5}$
3 Find the length BC. B $(3, -1) = (x_2, y_2)$ C $(-1, -3) = (x_1, y_1)$	$BC = \sqrt{[3-(-1)]^2 + [-1-(-3)]^2} = \sqrt{(4)^2 + (2)^2} = \sqrt{20} = 2\sqrt{5}$
4 Find the length AB. A $(1, 1) = (x_1, y_1)$ B $(3, -1) = (x_2, y_2)$	$AB = \sqrt{[3-(1)]^2 + [-1-(1)]^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$
5 State your proof.	Since $AC = BC \neq AB$, triangle ABC is an isosceles triangle.

Example 2

A triangle has vertices at P (0,-3), Q(-6,4) and R (5,1). Find the perimeter of the triangle to the nearest tenth of a unit and classify it



For the perimeter of ΔPQR, we must find the distances of $\overline{PQ}$, $\overline{QR}$ and $\overline{RP}$.

$$d_{\overline{PQ}} = \sqrt{(-6-0)^2 + (4--3)^2} = \sqrt{(-6)^2 + 7^2} = \sqrt{36+49}$$
$$d_{\overline{PQ}} = \sqrt{85}$$

$$d_{\overline{QR}} = \sqrt{(5--6)^2 + (1-4)^2} = \sqrt{11^2 + (-3)^2} = \sqrt{121+9}$$
$$d_{\overline{QR}} = \sqrt{130}$$

$$d_{\overline{RP}} = \sqrt{(0-5)^2 + (-3-1)^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25+16}$$
$$d_{\overline{RP}} = \sqrt{41}$$

$$Perimeter = \sqrt{85} + \sqrt{130} + \sqrt{41}$$

$$Perimeter \approx 27.0 \text{ units}$$

Since all three sides of the triangle are different in length, it is a **SCALENE TRIANGLE**.

Class Activity

Prove that the points A (0, −3), B (−2, −1) and C (4, 3) are the vertices of an isosceles triangle

Worksheet

Strand 4 - Graphs

Sketch the following graph:

(a) $x^2 + y^2 = 6$

(b) $2x^2 + 2y^2 = 50$

(c) $(x - 3)^2 + (y)^2 = 4$

(d) $y = \left(\frac{1}{5}\right)^x$

Strand 5: Coordinate Geometry

1. Find the distance between the points given below:
   (a) $(2, -8), (1, -7)$
   (b) $(3, 4), (-3, 7)$
   (c) $(1, 6), (-3, -4)$
2. Prove that the points P (4, –1), Q (5, 6) and R (1, 3) are the vertices of an isosceles triangle.
3. Show that the points D (1, 2), E (–6, 4) and F (5, –8) form a scalene triangle.
4. Show that the points A (7, 5), B (2, 3) and C (6, –7) are the vertices of a right triangle.