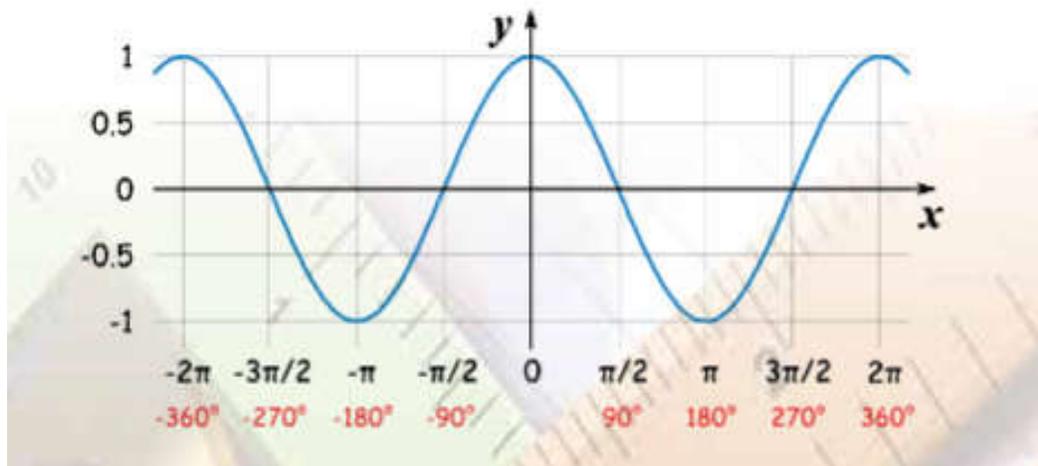


Ratu Navula College
Year 11 Applied Mathematics Lesson Notes – Week 6

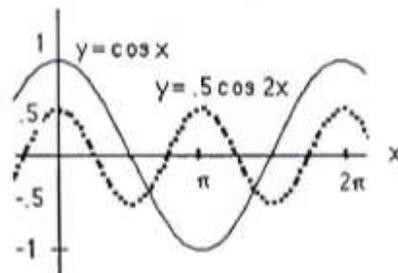
Lesson 51**Strand 6 : Trigonometry****Sub- Strand 6.1 : Trigonometric Relations****Learning Outcome : Sketch cosine graph.****Cosine Graph**

$$(y = a \cos bx)$$

Basic Cosine Graph**Example 1**Graph $y = \frac{1}{2} \cos 2x$

amplitude = $|a| = \frac{1}{2}$ The function has extreme values of $y = \frac{1}{2}$ and $y = -\frac{1}{2}$.

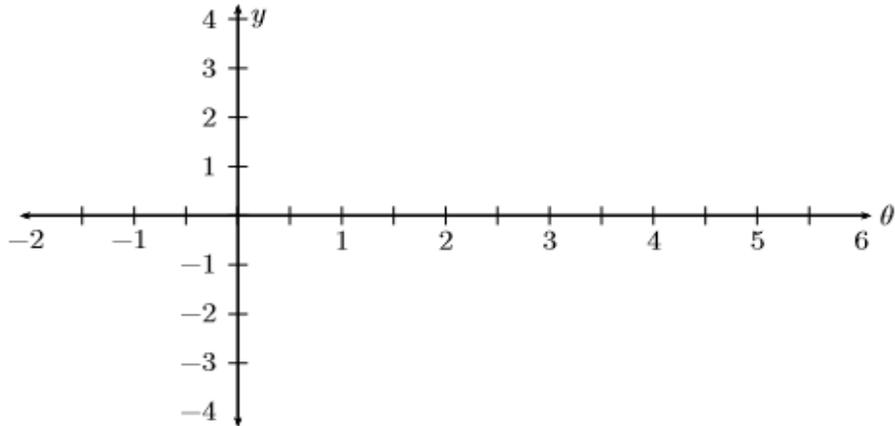
period = $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$ One complete cycle occurs between 0 and π .



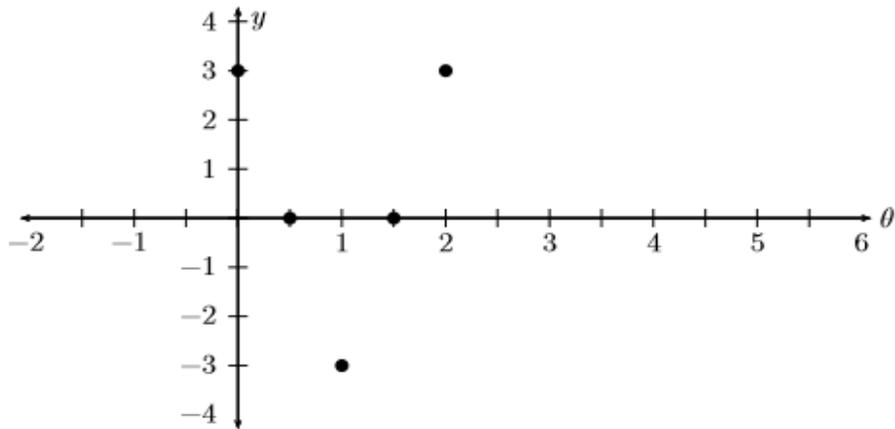
Example 2

Sketch the graph of $y = 3 \cos(\pi x)$

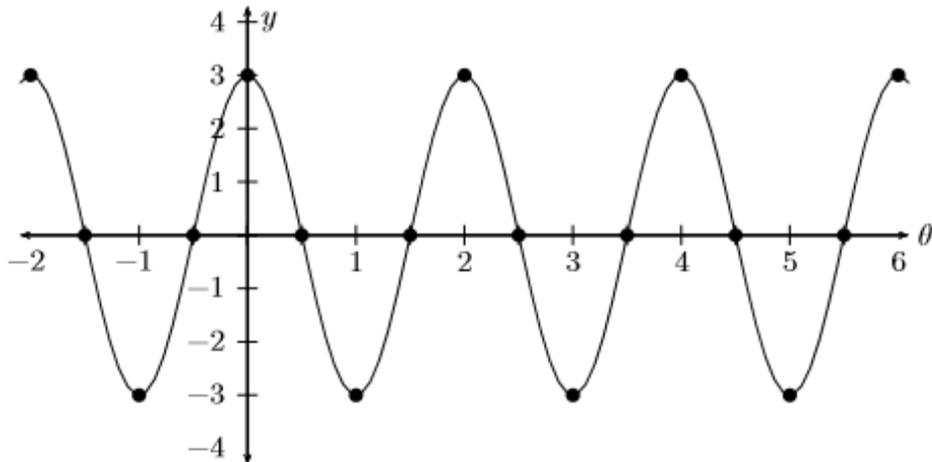
1. period = $\frac{2\pi}{\pi} = 2$. The amplitude is 3.
2. Label axes conveniently.



3. Mark zeros, maxima, and minima for one period (0 to 2).



4. Fill in remaining zeros, maxima, and minima. Draw a smooth curve.



Class Activity

1. Sketch the following graphs:

(i) $y = 3 \cos 4x$

(ii) $y = 4 \cos x$

Lesson 52

Strand 6 : Trigonometry

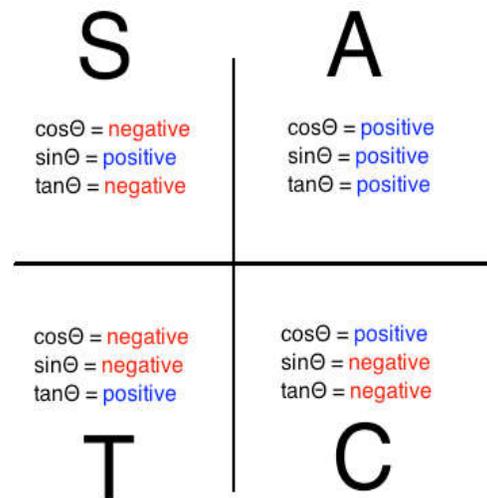
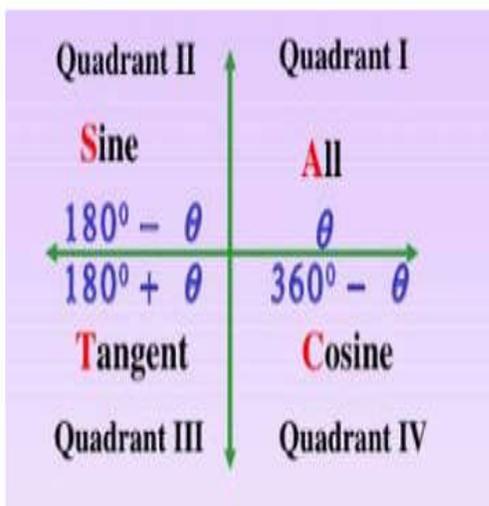
Sub- Strand 6.1 : Trigonometric Relations

Learning Outcome : Solve the trigonometry equation using the quadrant rules

When solving any trigonometric equation, emphasis must be given to the angle, θ , which can be either in **degrees or radians**.

To **solve for θ** , follow solving an algebraic equation:

- The value consisting θ , to be removed last.
- Do opposite operation on both sides of the equation till you reach the trigonometric expression containing sine, cosine or tangent.
- At this point in time, keep in mind that there will be at least two angles, within 0° to 360° or 2π radians.'
- If Trig expression is positive, then you will directly get the acute angle θ_1 . If Trig expression is negative, then ignore the negative sign to get the acute angle and use this to find the angle θ_1 .
- Use **quadrants** to find the other angle θ_2 . Angles will be considered from the positive x – axis.



Example 1: Solve $\tan \theta - 1 = 0, 0 \leq \theta \leq 2\pi$

^{Last}
 $\tan \theta - 1 = 0, 0 \leq \theta \leq 2\pi$ means that angle to be between $0 - 2\pi$

$$\tan \theta - 1 + 1 = 0 + 1,$$

$$\tan \theta = 1$$

$$\theta_1 = \tan^{-1} 1$$

$$\theta_1 = 45^\circ$$

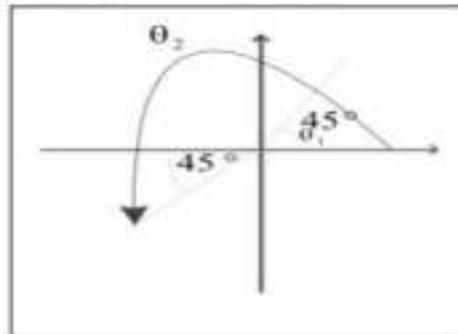


We reached at the trig expression: Consider the two quadrants. Find the acute angle in Q I. Note that calculator Mode to be in degrees.

- Use quadrants to find the angle θ_2 . Consider sign (+) of 'tan', that is in Q III

$$\begin{aligned} \theta_2 &= 180 + \theta_1 \\ &= 180 + 45 \\ &= 225^\circ \end{aligned}$$

$$\theta = 45^\circ, 225^\circ \text{ or } \theta \in \{45^\circ, 225^\circ\}$$



Example 2: Find the solution set for $2\cos \theta + \sqrt{3} = 0, 0^\circ \leq \theta \leq 360^\circ$

$2\cos \theta + \sqrt{3} = 0, 0^\circ \leq \theta \leq 360^\circ$ Means that angle to be between $0 - 360^\circ$

Last

$$2\cos \theta + \sqrt{3} - \sqrt{3} = 0 - \sqrt{3}$$

$$\frac{2\cos \theta}{2} = \frac{-\sqrt{3}}{2}$$

$$\cos \theta = \frac{-\sqrt{3}}{2}$$

We reached at the trig expression: Consider the two quadrants. But before that, find the acute angle by ignoring the negative sign (-). Note that calculator Mode to be in degrees.

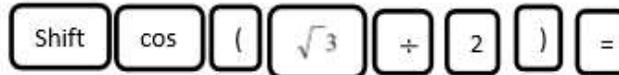
- **Acute angle:**

$$\theta = \text{Cos}^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$$

$$\alpha = 30^\circ$$



When dealing with surds,
press the division sign (+), that is Press

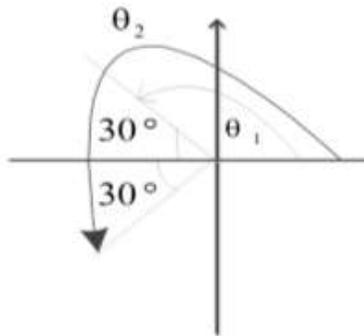


- Use quadrants to find the angles θ_1 and θ_2 . Consider negative sign (-) of Cos, that is in Q II / III

$$\theta_1 = 180 - 30 = 150^\circ$$

$$\theta_2 = 180 + 30 = 210^\circ$$

$$\theta = 150^\circ, 210^\circ \text{ or } \theta \in \{150^\circ, 210^\circ\}$$



EXAMPLE 3: Solve the trigonometric equation $\sin(x + 30^\circ) = 0.4$, where $-180^\circ \leq x \leq 180^\circ$.

Acute angle:

$$\sin(x + 30^\circ) = 0.4$$

$$(x + 30^\circ) = \sin^{-1} 0.4$$

$$\alpha = 23.58^\circ$$



Press

It already has trig expression:

Consider the two quadrants. Note that
calculator Mode to be in degrees.



- Use quadrants to find the angles θ_1 and θ_2 . Consider positive sign (+) of sin, that is in Q I / II

$$\theta_2 = 180 - \alpha = 180 - 23.58 = 156.42^\circ \quad \alpha = \theta_1 = 23.58^\circ$$

QI & II:

$$QI: x + 30^\circ = 23.58,$$

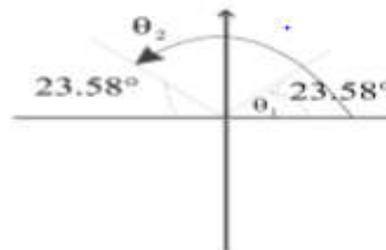
$$x + 30^\circ = 23.58,$$

$$x = -6.42^\circ$$

$$QII: x + 30^\circ = 156.42$$

$$x = 126.42^\circ$$

$$\therefore x \in \{-6.42^\circ, 126.42^\circ\}$$



Exercise

Solve the following trigonometric equation :

(a) $2 \cos \theta + \sqrt{3} = 0$ for $0^\circ \leq \theta \leq 360^\circ$

(b) $2 \sin \left(x + \frac{\pi}{4} \right) = 1$ where $0 \leq x \leq 2\pi$

(c) $2 \cos 2x = \sqrt{3}$ for $0 \leq x \leq 2\pi$

Lesson 53**Strand 7 : Statistics****Sub- Strand 6.1 : Statistics**

Learning Outcome : Define statistical terms (data, statistics, population, random sample, random numbers, tallying, frequency, sampling and biased)

STATISTICAL TERMS

1. **Data** – information gathered through surveys. Can be discrete (whole numbers) or continuous (decimal points)
2. **Qualitative data** – descriptive data for example “It was a fun game”
3. **Quantitative data** – is a numerical information. Two types are :
 - **Discrete data** – whole number values (usually obtained from counting)
 - **Continuous data** – whole numbers as well as fractions (usually obtained by measurement)
4. **Statistics** – branch of mathematics that deals with scientific methods of collecting information (data), organising and presenting it, analysing and then making relevant conclusions.
5. **Population** – represents the entire group under study
6. **Sample** – smaller group taken from within the population on which the actual study is done
 - **Random sample** – a sample selected where each element has an equal chance of selection, that is, fair or unbiased sample
7. **Random numbers** – a set of numbers consisting of digits 0 to 9. These numbers exist in the random number table (Page 36 of Etons Table). The distribution of numbers does not have a pattern and each digit has the same probability of occurrence at any spot in the table
8. **Tallying** – score or amount
9. **Frequency** – number of observations in or category
10. **Sampling** – selecting people/objects from a population in order to test the population for something.
11. **Biased - sample** is collected in such a way that some members of the intended population are less likely to be included than others.

12. **Census** – is when the data is collected from every member of the group
13. **Sample** – when the data is collected from a selected members or the elements of the group.

Steps involved in conducting a survey:

- Create the question
- Ask the question
- Tally the result
- Present the result

Exercise

1. Differentiate between discrete and continuous data?
2. Differentiate between census and sample?
3. Discrete and Continuous data refers to _____.

Lesson 54

Strand 7 : Statistics

Sub- Strand 6.1 : Statistics

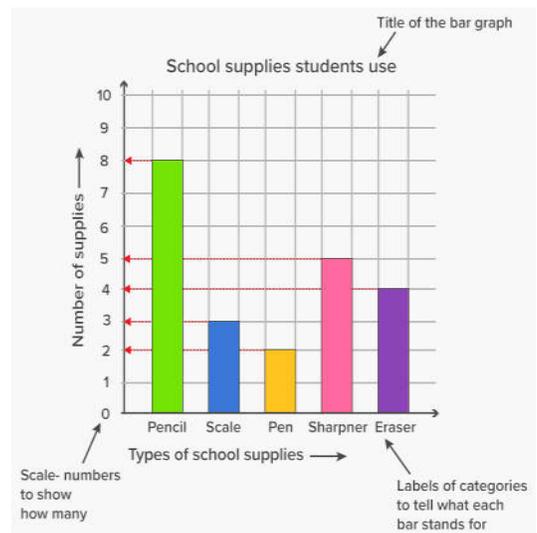
Learning Outcome : Differentiate between grouped and ungrouped data.

UNGROUPED DATA PRESENTATION

1. TABLE

Favourite Fruits	Number of Students
Grapes	3
Apple	3
Banana	4
Pineapple	2
Orange	1
Watermelon	1
Kiwi fruit	4
Mango	6
TOTAL	24

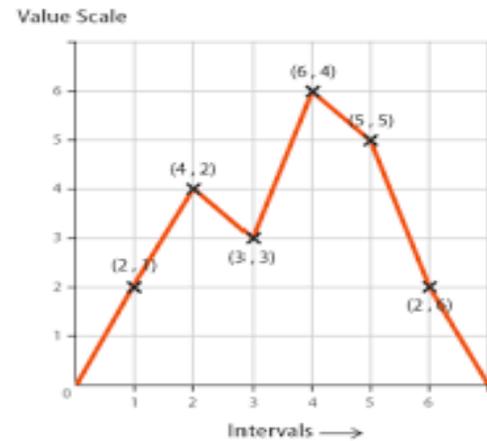
2. BAR GRAPH



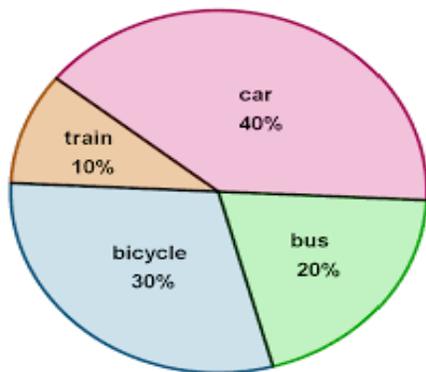
3. PICTOGRAPH

Mode of transport	Number of students
Bus	☺☺☺☺☺☺☺☺
Car	☺☺☺☺
Walking	☺☺☺☺☺☺☺
Bicycle	☺☺☺
Key : ☺ Represents 3 children	

4. LINE GRAPH



5. PIE CHART



$$\text{Angle} = \frac{\text{No. of students}}{\text{Total students}} \times 360^\circ$$

GROUPED DATA (CONTINUOUS)

- When data is too large, it is divided into classes or categories, that is, data is grouped
- Grouped data has class intervals instead of individual scores
- Class mark is the midpoint of the class interval ($40 \leq \text{mass} \leq 45$) 45 is not included

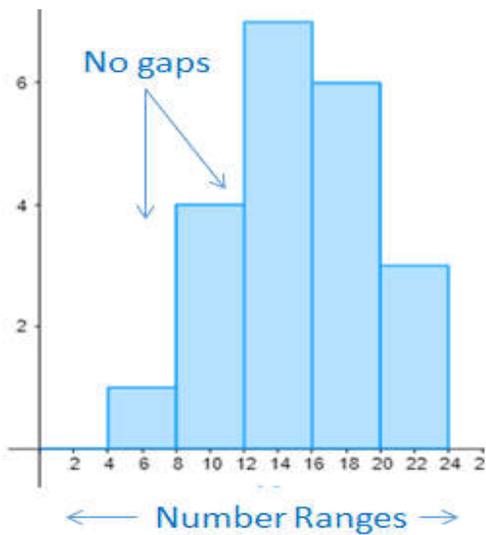
PRESENTATION OF GROUPED DATA

1. HISTOGRAM

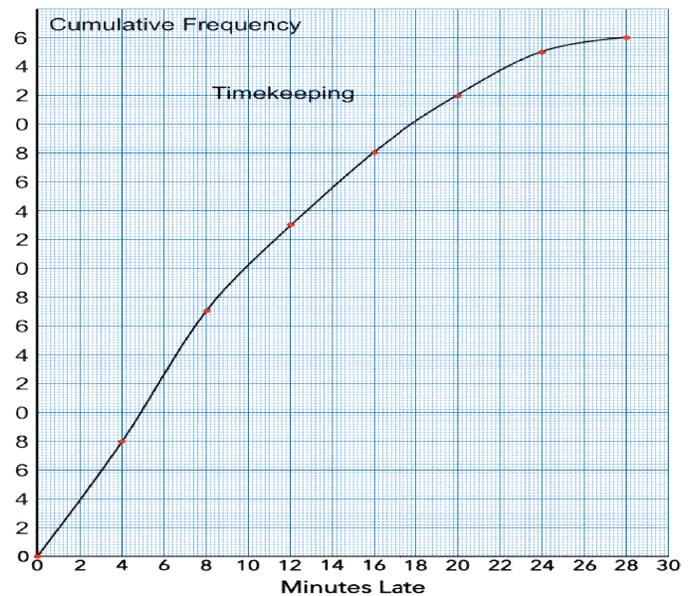
- Grouped data that is continuous can be displayed on bar graphs called histograms
- Bars on the histogram touch because the data is continuous

- The class mark will be the centres of the bars
 - Width is determined by the size of the class interval
 - The area of each rectangle is proportional to the frequency
2. **CUMULATIVE FREQUENCY** - is the total frequency less than or equal to the given class interval, that is, sum of all frequencies at that interval and before that interval

HISTOGRAM



CUMMULATIVE FRQUENCY



Every graph must :

- Have a title
- Be labeled properly
- Axis should be equally distributed or scaled.

Exercise

1. What is the main difference between histogram and bar graph?
2. What is the difference between grouped and ungrouped data?

Lesson 55**Strand 7 : Statistics****Sub- Strand 6.1 : Statistics**

Learning Outcome : Calculate measure of central tendency and advantages and disadvantages of different measures of tendency.

MEASURES OF CENTRAL TENDENCY

✓ Are used to measure the location of central value in the data.

1. MEAN

- Commonly known as average
- **Symbol:** \bar{x}

$$\text{Ungrouped data: } \bar{x} = \frac{\sum \text{scores } (x)}{\text{Number of scores } (n)}$$

$$\text{Grouped data: } \bar{x} = \frac{\sum fx}{\sum f}$$

2. MEDIAN

- The middle value when scores are arranged in ascending or descending order
- Divides the score into two equal parts
- For frequency distribution, the median corresponds to 50th percent of the frequency

3. MODE

- Most common score or the number with highest frequency

Advantages and Disadvantages for Mean, Median, and Mode

Measure of Central Tendency	Advantage	Disadvantage
Mean	<ul style="list-style-type: none"> ■ Easy to compute ■ Takes all data values into account ■ Reliable 	<ul style="list-style-type: none"> ■ Influenced by extreme values
Median	<ul style="list-style-type: none"> ■ Easy to compute ■ Not influenced by extreme values ■ Can be used with non-numerical data 	<ul style="list-style-type: none"> ■ Difficult to rank large number of data values
Mode	<ul style="list-style-type: none"> ■ Easily found ■ Not influenced by extreme values ■ Can be used with non-numerical data 	<ul style="list-style-type: none"> ■ Can't always locate just one mode

Example 1

Find the mean for the following :

(a) $\{2, 4, 6, 8, 10\}$

$$\begin{aligned}\bar{x} &= \frac{2 + 4 + 6 + 8 + 10}{5} \\ &= \frac{30}{5} \\ &= 6\end{aligned}$$

(b) $\{3, 6, 7, 8, 12\}$

$$\begin{aligned}\bar{x} &= \frac{3 + 6 + 7 + 8 + 12}{5} \\ &= \frac{36}{5} \\ &= 7.2\end{aligned}$$

Example 2

The frequency table shows the age group of students in a class. Calculate the mean age of the class.

Age (x)	16	17	18	19
Frequency (y)	8	3	4	20

Solution

Age(x)	Frequency(y)	x . f
16	8	$16 \times 8 = 128$
17	3	$17 \times 3 = 51$
18	4	$18 \times 4 = 72$
19	20	$19 \times 20 = 380$
Total	35	631

$$\begin{aligned}\bar{x} &= \frac{\sum fx}{\sum f} \\ &= \frac{631}{35} \\ &= \underline{18.03 \text{ years}}\end{aligned}$$

Example 3

The mean weight of a group of 6 students is 62kg . Another student whose weight is 52kg is to be included in the group. Find the mean weight of 7 students?

Solution

mean = 62 kg

n = 6

$$\bar{x} = \frac{\sum \text{scores } (x)}{\text{Number of scores } (n)}$$

$$62 = \frac{x}{6}$$

$$x = 62 \times 6$$

$$x = 372 \text{ kg}$$

if another students weight of 52 kg is added then the total will be increased to 52kg as well as the number of students be 7 now.

$$\bar{x} = \frac{\sum \text{scores } (x)}{\text{Number of scores } (n)}$$

$$= \frac{372+52}{7}$$

$$= 60.6 \text{ kg}$$

(new mean weight for 7 students)

Exercise

1. Find the mean for the following data :

(a) {15 , 4 , 7 , 21 , 41 , 87 }

(b) {-1 , 56 , 0 , 31 , 75 , 30 }

2. Use the following data to find the mean.

x	1	2	3	4	5
y	8	4	4	7	1

WORKSHEET 6**2016**

Sketch the **graph** of the function $y = -2 \cos 2\theta$ for $0^\circ \leq \theta \leq 360^\circ$

2015

Sketch the graph of the function $y = 3 \sin 2\theta$ for $0^\circ \leq \theta \leq 180^\circ$.

2018

Solve the **trigonometric equation** $2 \sin \theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$

2017

Solve the **trigonometric equation** $2 \tan \theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$

2016

Solve $2 \sin \theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$.

2015

Solve $2 \cos \theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$

2018

A **trigonometric graph** has the equation $y = -2 \cos 2\theta$

- (i) State the **amplitude** of this graph.
- (ii) What is the **y-intercept** of this graph?
- (iii) Sketch the graph for $0^\circ \leq \theta \leq 360^\circ$

2017

A **trigonometric graph** has the equation $y = 3 \sin 2\theta$

- (i) State the **amplitude** of this graph.
- (ii) What is the **period** of this graph?
- (iii) Sketch the graph for $0^\circ \leq \theta \leq 360^\circ$