BA PROVINCIAL FREE BIRD INSTITUTE

YEAR 12 TERM 1 END EXAMINATION 2021

MATHEMATICS
QUESTION & ANSWER BOOKLET

Name: _____
Year: _____

Time Allowed: Three Hours
(An extra ten minutes allowed for reading this paper)

INSTRUCTIONS

1. Write **all** your answers in the appropriate places provided in this paper.
2. Write your name on front page of this paper.
3. You may use calculators provided they are silent, battery operated and non-programmable.
4. There are two sections for each question in this paper. **Both sections are compulsory.**
5. Unless otherwise stated, all rounding off should be corrected to **two decimal places.**
6. Rounding off decimal answers should be done only at the final step.

SUMMARY OF QUESTIONS

There are three questions. All are compulsory.

QUESTION	STRANDS	TOTAL MARK	SUGGESTED TIME
1	Basic Math	29	50 minutes
2	Algebra	42	80 minutes
3	Graphs	29	50 minutes
	Total	**100**	**180 minutes**

Mark Gained:

QUESTION 1 BASIC MATHEMATICS [29 Marks]

There are two parts to this section. Answer both parts.

PART I: MULTIPLE CHOICE QUESTIONS [7 Marks]

Circle the letter of your best choice.

1. The table below shows the results of **t** = [w, x, y, z] under operation @.

@	w	x	y	z
w	y	z	w	X
x	z	w	x	y
y	w	x	y	z
z	x	y	z	w

The inverse of y is

A. w B. x C. y D. z

1	
0	
NR	

2. A credit card will attract a high interest rate. This is because

A. It does not have a bank account with it.
B. The customer can pay it off at any time.
C. The credit card is usually only for small amounts of credit.
D. Credit cards are unsecured lending.

1	
0	
NR	

3. For the binary operation @ on the set M, if a @ (b @ c) = (a @ b) @ c is true for all values of a, b and c on M, then the operation @ is said to be:

A. closed B. a group C. associative D. commutative

1	
0	
NR	

4. When simplified $(4^3)^2$ is equal to:

$(ab)^m = a^m b^m$

A. 4^5 B. 4^6 C. 4^9 D. 4^{32}

1	
0	
NR	

5. If $a @ b = a^2 - b$, then 3 @ 4 is equal to

A. 5 B. 4 C. 3 D. 2

1	
0	
NR	

6. When expressed in base index form, $\log_a b = c$ is

$y = b^x \leftrightarrow \log_b y = x$

A. $a^b = c$ B. $a^c = b$ C. $c^b = a$ D. $b^c = a$

1	
0	
NR	

7.	$\left(\frac{16}{4}\right)^{-2}$ when simplified is equal to A. $\frac{1}{16}$ B. $\frac{4}{2}$ C. $\frac{2}{4}$ D.16	1 ☐ 0 ☐ NR ☐

PART II: SHORT ANSWER QUESTIONS **(22 Marks)**

Show working on the spaces provided.

8.	Evaluate $\frac{3^{n-2}}{3^n}$ $a^m \div a^n = a^{m-n} : a \neq 0$	1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
9.	The table on operation @ is shown below Is the system shown a group? Show.	4 ☐ 3½ ☐ 3 ☐ 2½ ☐ 2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐

@	1	3	5	7
1	1	3	5	7
3	3	1	7	5
5	5	7	1	3
7	7	5	3	1

10.	Expand and simplify(give the exact value): $\sqrt{2}(\sqrt{8}-1)$	1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
11.	Express $2\log x + \frac{1}{4}\log 16 - \log \frac{x}{2}$ as the logarithm of a single number $\log ab = \log a + \log b$ $\log \frac{a}{b} = \log a - \log b$ $\log x^n = n \log x$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
12.	Simplify (i) $\frac{(x-3)^{\frac{1}{2}}}{(x-3)^{\frac{1}{2}}}$ (ii) $6 + \sqrt{5} + 3\sqrt{5} - 5\sqrt{5}$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
14.	Solve: (i) $m^5 = 3125$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐

	(ii) $\log_2 25 = x$ $y = b^x \leftrightarrow \log_b y = x$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
15.	The cash price of a bike is \$220. The hire purchase price is \$300. If the deposit is 10% followed by 10 equal monthly instalments. (i) Calculate the deposit	1 ☐ 0 ☐ NR ☐
	(ii) Calculate the total instalment	1 ☐ 0 ☐ NR ☐
	(iii) Calculate the monthly payment	1 ☐ 0 ☐ NR ☐

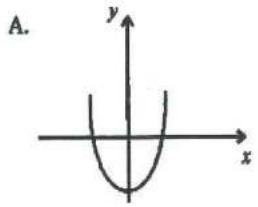
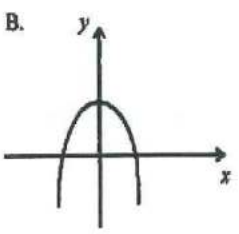
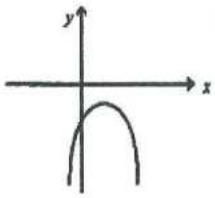
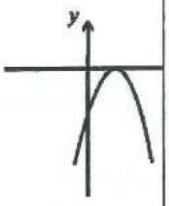
16.	Rationalize the denominator of the following: $\frac{7}{3-\sqrt{2}}$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
17.	Simplify: $\frac{3\times 9^{2x-1}}{3^{5x}}$ 1. $a^m \times a^n = a^{m+n}$ 2. $a^m \div a^n = a^{m-n}$; $a \neq 0$ 3. $(a^m)^n = a^{mn}$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐

QUESTION 2: **ALGEBRA** **[42marks]**

There are two parts to this section. Answer both parts.

PART I: MULTIPLE CHOICE QUESTIONS **(7 Marks)**

1.	The fifth term of the arithmetic sequence { 3x, 6x, ... } is $T_n = a+(n-1)d$ A. 15x B. 12x C.10x D. 48x	1 ☐ 0 ☐ NR ☐

2.	Which of these graphs has no real root? A.  B.  C.  D. 	1 ☐ 0 ☐ NR ☐
3.	What is the value of $\sum_{k=3}^{5} k+1$? A. 8 B. 15 C.9 D. 20	1 ☐ 0 ☐ NR ☐
4.	Which of the following is equivalent to $\frac{x-2}{12-3x^2}$? A. -3 (2 + x) B. $\frac{-1}{6+x}$ C. $\frac{-1}{6+3x}$ D. $\frac{x}{6+3x^2}$	1 ☐ 0 ☐ NR ☐
5.	The solution set to the equation $(x-2)^2 = 16$ is A. { 6,-2 } B. { -6,2 } C. { 4,2 } D. { -4,2 }	1 ☐ 0 ☐ NR ☐
6.	$9x^2 - 36$ when **completely factorized** equals: A. $9(x+2)(x-2)$ B. $9(x+4)(x-4)$ C. $9(x^2-4)$ D. $9(x+2)(x+2)$	1 ☐ 0 ☐ NR ☐
7.	The solution to the inequality $3 - 2x > -9$ is, A. $x > 6$ B. $x < 6$ C. $x > 3$ D. $x < 3$	1 ☐ 0 ☐ NR ☐

PART II: SHORT ANSWER QUESTIONS (35 Marks)

Show working on the spaces provided.

8. Solve:

(i) $\frac{3}{x} = \frac{2}{x-1}$

2	
1½	
1	
½	
0	
NR	

Simplify:

(ii) $\frac{x^3 + 4x^2}{8x^2} \div \frac{x+4}{16}$

2	
1½	
1	
½	
0	
NR	

9. (i) Show that (x - 3) is a factor of $f(x) = x^3 + 2x^2 - 9x - 18$.

1	
0	
NR	

	(ii) Hence factorize f(x) = $x^3 + 2x^2 - 9x - 18$ completely.	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
10.	Calculate the value of $\sum_{n=4}^{6} n^3$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
11.	The first term of a geometric sequence is 21 and the ratio is $\frac{1}{3}$. Find the sum to infinity of this sequence. $S_\infty = \frac{a}{1-r}$;if $\lvert r \rvert < 1$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐

12. The series $\{\frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \ldots\}$ is geometric.

(i) Write down the common ratio.

$r = t_2 \div t_1$

1	
0	
NR	

(ii) Find the value of T_4.

$$T_n = ar^{n-1}$$

1	
0	
NR	

(iii) Use an appropriate formula to determine the sum of first 8 terms.

$$S_n = \frac{a(1-r^n)}{1-r}; r \neq 1$$

2	
1½	
1	
½	
0	
NR	

13. Make x the subject of the formula

$$y = \frac{2x+3}{x-1}$$

2	
1½	
1	
½	
0	
NR	

14.	The first, fifth and last terms of an arithmetic progression are 3, 23 and 103 respectively. How many terms are there in the series? $T_n = a + (n-1)d$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
15.	Use the **quadratic formula** to solve the equation $2x^2+1 = 4x$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
16.	Find the discriminant and state the nature of the root of the quadratic expression $f(x) = 2x^2 - x + 1$.	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐

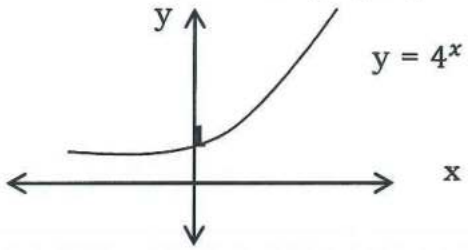
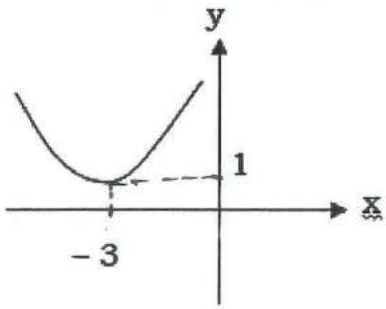
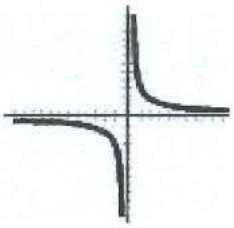
17.	A quadratic expression is given as $f(x) = \mathbf{a}x^2 - 4x + 2$. For what value of **a** will the roots of f(x) be real and equal? (Hint : $b^2 - 4ac = 0$)	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
18.	Solve the equation $\frac{x+4}{2} + \frac{2x}{3} = 2$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
19.	(i) Evaluate $\sum_{i=2}^{5} -3n^2 + 2$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐

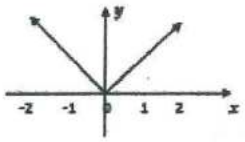
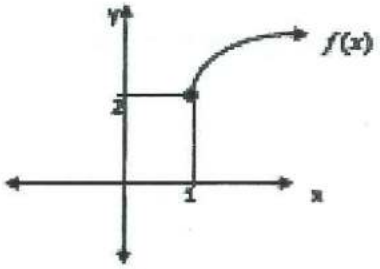
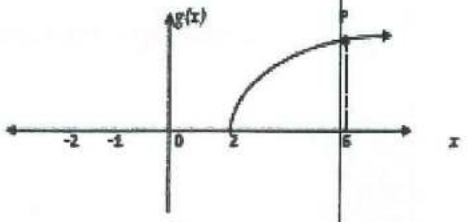
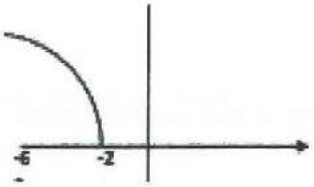
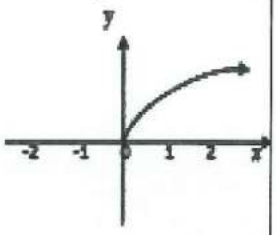
19.	Solve the following inequation. (ii) $x^2 + 2x - 24 > 0$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
20.	Find the remainder when $x^2 - 3x + 6$ is divided by $(x - 2)$	1 ☐ 0 ☐ NR ☐
21.	Find the value of k for which $(x - 1)$ is a factor of $x^3 - 4kx + 5$	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
22.	A square hen house has an area of 169 square feet. What is the length of the hen house?	1 ☐ 0 ☐ NR ☐

QUESTION 3: **GRAPHS** **[29marks]**

There are two parts to this section. Answer both parts.

PART I: MULTIPLE CHOICE QUESTIONS **(6 Marks)**

1.	Which of the following is the general form of the equation of a circle with center (a,b) : $(x-a)^2+(y-b)^2=r^2$ A. $y=\frac{ax+b}{cx+d}$ B. $(x+a)^2+(y+b)^2= r^2$ C. $x^2+y^2=r^2$ D. $(x-a)^2+(y-b)^2= r^2$	1 / 0 / NR
2.	State the domain of the exponential graph given below.  A. $x>1$ B. $x<1$ C. $x \in R$ D. $y \geq 0$	1 / 0 / NR
3.	Which of the following equations best represents the graph shown? A. $y=(x+3)^2+1$ B. $y=(x-3)^2-1$ C. $y=x^2+3$ D. $y=x^2-3$ 	1 / 0 / NR
4.	Which of the following best represents the equation of the graph shown:  A. $y=\frac{1}{x}$ C. $y=\frac{-1}{x}$ B. $y=x^4$ D. $y=\frac{x}{2}$	1 / 0 / NR

5.	Which of the following best describes the equation of the graph shown?  A. $y = \|x\|$ B. $y = x^2$ C. $y = x^{\frac{1}{2}}$ D. $y = \sqrt{x}$	1 ☐ 0 ☐ NR ☐
6.	Which of the diagram below best shows the graph of $g(x) = \sqrt{x-1} + 2$ A.  C.  B.  D. 	1 ☐ 0 ☐ NR ☐

PART II: SHORT ANSWER QUESTIONS **(23 marks)**

Show working on the spaces provided.

8.	Sketch the graph of $y = (x - 2)(x + 1)$ showing x and y intercepts and turning point clearly.	3 ☐ 2½ ☐ 2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐

9.	For the function $g(x) = \frac{3x+6}{x-2}$ i) Find the x and y – intercepts of g(x).	1 | 0 | NR
	ii) Find the vertical and horizontal asymptotes of g(x).	2 | 1½ | 1 | ½ | 0 | NR
9.	(iii) Sketch g(x) clearly showing the intercepts and asymptotes.	3 | 2½ | 2 | 1½ | 1 | ½ | 0 | NR

10.	Function f(x) is defined as : f(x) = - (x-1)(x-2)(x+3) i) Find the x-intercept and y intercept of the graph of f(x). ii) Sketch the graph showing all intercepts.	1 ☐ 0 ☐ NR ☐ 2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐
11.	A function is given as $(x-2)^2 + (y+1)^2 = 16$. (i) Find the centre of the circle. (ii) Find the radius of the circle.	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐ 1 ☐ 0 ☐ NR ☐

12. A function is defined as $f(x) = 3^x$

(i) Find the coordinates of the y-intercept.

1	
0	
NR	

(ii) Sketch the graph of $f(x)$ and label it clearly.
(Graph paper not required.)

2	
1½	
1	
½	
0	
NR	

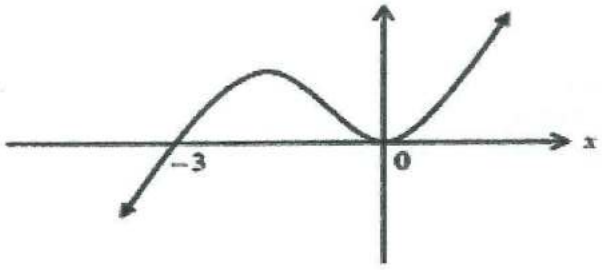
Another function is defined as $g(x) = \log_3 x$

(iii) On the same pair of axes, sketch the graph of the function $g(x)$, showing the x-intercept clearly.

2	
1½	
1	
½	
0	
NR	

12. (iv) Describe fully the transformation that maps the graph of function $f(x)$ onto the graph of function $g(x)$.

1	
0	
NR	

13.	Give the equation of the cubic function shown below.  	2 ☐ 1½ ☐ 1 ☐ ½ ☐ 0 ☐ NR ☐

THE END