

JASPER WILLIAMS HIGH SCHOOL
YEAR 12 ANNUAL EXAMINATION 2020

MATHEMATICS

Time Allowed: 3 Hours

(An extra 10 minutes is allowed for reading this paper)

INSTRUCTIONS

1. Write **all** your answers in the **Answer Book** provided. For **multiple – choice questions**, write the **letter** of the **best** answer in the space provided in the **Answer Book**.
2. Write your **Index Number** on the front page of the **Answer Book**.
3. If you use extra sheets of paper, be sure to clearly show the question number(s) being answered and staple each sheet securely in your **Answer Book** at the appropriate place.
4. You may use a calculator, provided it is silent, battery-operated and non-programmable.
5. Round off answers to 2 decimal places where applicable. Rounding off decimal answer should be done at the final step.
6. All questions **are compulsory**.

Note: The required formulae and table are given on **pages 18 – 20**.

SUMMARY OF QUESTIONS

STRANDS		MARKS	SUGGESTED TIME
1.	Basic Mathematics	12	22 minutes
2.	Algebra	18	32 minutes
3.	Graphs	12	22 minutes
4.	Coordinate Geometry	7	13 minutes
5.	Trigonometry	10	18 minutes
6.	Matrices and Transformation	7	13 minutes
7.	Statistics	6	10 minutes
8.	Probability	10	18 minutes
9.	Calculus	18	32 minutes
TOTAL		100	180 minutes

STRAND 1

BASIC MATHEMATICS

[12 marks]

- This strand has **8 Questions**
- **Choose** the best answer and write the **letter** of your choice for **Questions 1 - 3**.
- Show all working for **Questions 4 - 8**.

1. The expression $\log xy - \log z$ can be simplified to

A. $\log \frac{xy}{z}$

B. $\log \frac{z}{xy}$

C. $\log xyz$

D. $\log xy + z$

(1 mark)

2. $2 + 3$ in **modulo 4** is equal to

A. 0

B. 1

C. 2

D. 3

(1 mark)

3. When simplified $8 + \sqrt{7} + 6\sqrt{7}$ is equal to

A. $8 + 7\sqrt{7}$

B. $14 + 2\sqrt{7}$

C. $\sqrt{57}$

D. $\sqrt{28}$

(1 mark)

4. Simplify $\frac{7}{3-\sqrt{2}}$ by rationalizing the denominator

(2 marks)

5. Solve the equation $3^{x+1} = 9$

(1½ marks)

6. Simplify $\frac{4^{5x}}{2^x}$

(1½ marks)

7. Shane purchased a laptop on **hire purchase**. She paid \$100 deposit and 24 monthly payments of \$30. Calculate the **total** amount she paid for the laptop. (1 mark)
8. The table given below shows the set $\{0, 1, 2\}$ under the operation \star . Use the table to answer parts (a) - (c).

\star	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

- (a) What is the **identity element**? (1 mark)
- (b) Give the **inverse** of 1. (1 mark)
- (c) Evaluate: $(0 + 1) + 2$. (1 mark)

STRAND 2

ALGEBRA

[18 marks]

- This strand has **12 Questions**
- **Choose** the best answer and write the **letter** of your choice for **Questions 1 - 5**.
- Show all working for **Questions 6 - 12**.

1. The **solution** set of $(x - 2)(x + 3) = 0$ is

- A. $\{2, 3\}$
- B. $\{2, -3\}$
- C. $\{-2, 3\}$
- D. $\{-2, -3\}$

(1 mark)

2. If $f(x) = x^2 + x - 2$ is divided by $x - 1$, the **remainder** is

- A. 0
- B. 1
- C. 2
- D. 3

(1 mark)

3. The value of the summation given below is

$$\sum_{n=1}^{n=3} (n + 1)$$

- A. 2
- B. 3
- C. 4
- D. 9

(1 mark)

4. If $4m + k = 9$, then

- A. $m = \frac{9-k}{4}$
- B. $m = \frac{9+k}{4}$
- C. $m = \frac{9}{4} - k$
- D. $m = 9 - \frac{k}{4}$

(1 mark)

5. The **third** term of the **arithmetic sequence** $\langle 2x, 5x, \dots \rangle$ is
- A. $3x$
 - B. $7x$
 - C. $8x$
 - D. $11x$
- (1 mark)**
6. Use the **quadratic formula** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve $x^2 - 2x - 4 = 0$
- (2 marks)**
7. Consider the quadratic equation: $3x^2 - 4x + 5 = 0$.
- (a) Calculate the value of the **discriminant**. **(1 mark)**
 - (b) Hence, state the **nature** of the roots. **(1 mark)**
8. Simplify: $\frac{4x}{y} - \frac{x}{3} \div \frac{y}{3}$ **(2 marks)**
9. Solve: $\frac{x+1}{3} = \frac{2x+6}{8}$ **(2 marks)**
10. A geometric sequence is given as $100, 20, 4, \dots$
- Find the **sum to infinity**. **(1 mark)**
11. A polynomial function is given by: $f(x) = x^3 - 5x^2 - 2x + 24$.
- Given that $x + 2$ is one of the factors of $f(x)$, find the other two factors. **(2 marks)**
12. The first four terms of an arithmetic sequence are given as $\langle 2, 6, 10, 14, \dots \rangle$.
- Calculate:
- (a) The 6th term. **(1 mark)**
 - (b) The sum of the first 10 terms. **(1 mark)**

STRAND 3

GRAPHS

[12 marks]

- This strand has 7 Questions
- Choose the best answer and write the letter of your choice for Questions 1 - 4.
- Show all working for Questions 5 - 7.

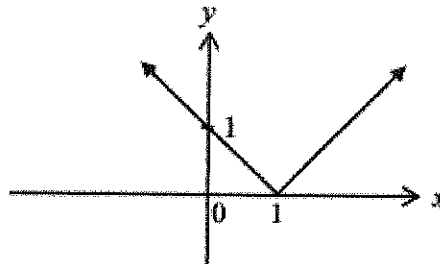
1. A relation is given by a set of ordered pairs: $\{(1, 2), (2, 4), (3, 6)\}$.

The **inverse** of this relation is

- A. $\{(2, 1), (4, 2), (6, 3)\}$
- B. $\{(1, 2), (2, 4), (3, 6)\}$
- C. $\{(-1, 2), (-2, 4), (-3, 6)\}$
- D. $\{(-1, -2), (-2, -4), (-3, -6)\}$.

(1 mark)

2. The equation of the **absolute value** function shown below is



- A. $y = |x| + 1$
- B. $y = |x| - 1$
- C. $y = |x - 1|$
- D. $y = |x + 1|$

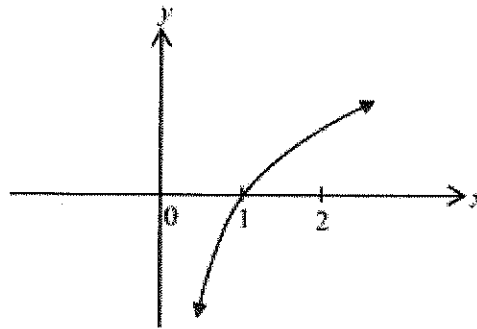
(1 mark)

3. The coordinates of the **centre** of the circle $(x - 3)^2 + y^2 = 4$ are

- A. (3, 0)
- B. (3, 4)
- C. (-3, 0)
- D. (-3, 4)

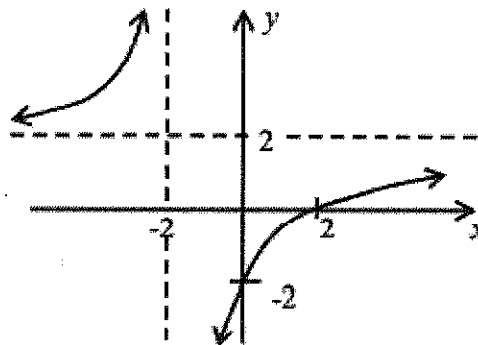
(1 mark)

4. Consider the graph of the function $y = \log_3 x$ shown below.



The **domain** of this function is

- A. $x > 0$
 B. $x \geq 0$
 C. $x = \{0, 1, 2\}$
 D. $x = \{0, 1, 2, \dots\}$
5. Find the **x coordinates** of the points of intersection of the functions
 $y = x + 5$ and $x^2 + y^2 = 25$
6. A hyperbolic function is given below.



- (a) Write down the **x and y intercepts** of this function. (1 mark)
 (b) Give the equations of the horizontal and vertical **asymptotes**. (1 mark)
 (c) Determine the equation of this hyperbola. (2 marks)
7. Sketch the graph of $y = (x + 2)(x - 1)(x - 3)$ clearly showing all intercepts. (2 marks)

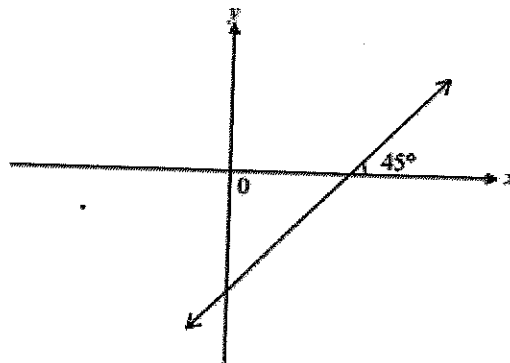
STRAND 4

COORDINATE GEOMETRY

[7 marks]

- This strand has 4 Questions
- Choose the best answer and write the letter of your choice for Questions 1 - 2.
- Show all working for Questions 3 - 4.

1. A line makes an angle of with the positive axis.



The gradient of this line is

- A. 1
- B. 2
- C. 3
- D. 4

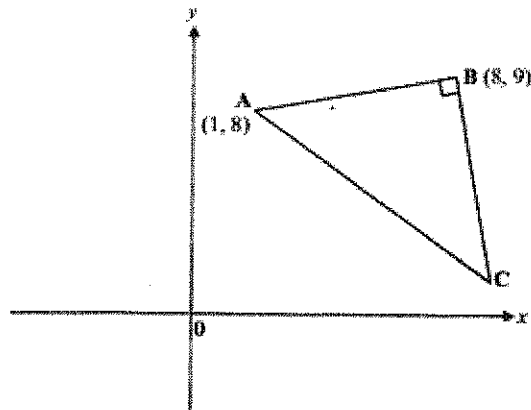
(1 mark)

2. If the lines $y = kx$ and $y = \frac{1}{4}x$ are perpendicular, then the value of k is

- A. 4
- B. $\frac{1}{4}$
- C. $-\frac{1}{4}$
- D. -4

(1 mark)

3. The diagram given below shows a **right-angled** triangle with vertices **A**, **B** and **C**.



The coordinates of point **A** = (1, 8) and **B** = (8, 9). **AB** is **perpendicular** to **BC**.

- (a) Calculate the **length** of **AB**. (1 mark)
- (b) Calculate the **gradient** of **AB**. (1 mark)
- (c) Hence, determine the **gradient** of **BC**. (1 mark)
4. Show that the following points are **collinear**: (1, 4), (4, 6) and (10, 10) (2 marks)

STRAND 5

TRIGONOMETRY

[10 marks]

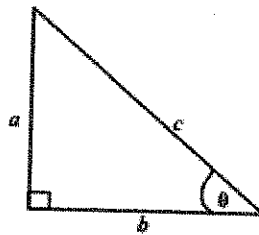
- This strand has **6 Questions**
- **Choose** the best answer and write the **letter** of your choice for **Questions 1 - 2**.
- Show all working for **Questions 3 - 6**.

1. How many radians make up a **full turn**?

- A. 2
 B. π
 C. 2π
 D. 360

(1 mark)

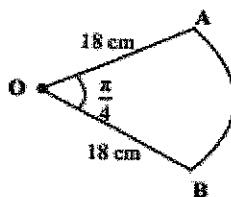
2. Which of the following expressions gives the size of angle θ ?



- A. $\tan^{-1}\left(\frac{a}{b}\right)$
 B. $\tan^{-1}\left(\frac{b}{a}\right)$
 C. $\sin^{-1}\left(\frac{b}{c}\right)$
 D. $\cos^{-1}\left(\frac{a}{c}\right)$

(1 mark)

3. A **sector** with centre **O**, radius 18 cm and angle $\frac{\pi}{4}$ radians is shown below.



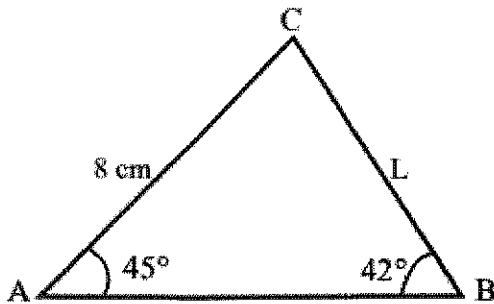
(a) Calculate the **length** of arc **AB**

(1 mark)

(b) Calculate the **area** of the sector.

(1 mark)

4. Calculate the value of the side marked L in the triangle below.



(2 marks)

5. Sketch the graph of $y = 3\sin(\theta + 90^\circ)$ for $0^\circ \leq \theta \leq 360^\circ$

(2 marks)

6. Solve the trigonometric equation $\sqrt{3}\tan\theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$

(2 marks)

STRAND 6

MATRICES AND TRANSFORMATION

[7 marks]

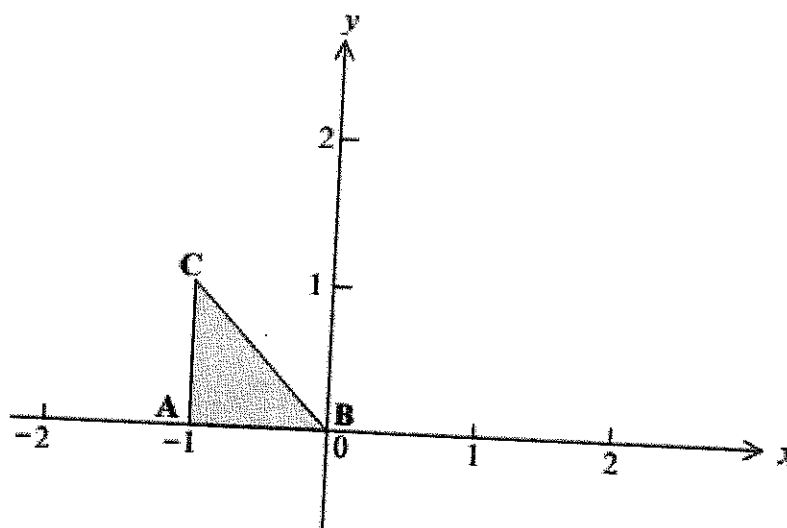
- This strand has 2 Questions
- Choose the best answer and write the letter of your choice for Question 1.
- Show all working for Question 2 (a).

1. A square is transformed by the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$. By what factor does the area of the square increase?

- A. 3
- B. 6
- C. 9
- D. 81

(1 mark)

2. Triangle ABC shown below is transformed by the matrix $N = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.



The coordinates of $B' = (0, 0)$ under the transformation by matrix N .

- (a) Find the coordinates of A' and C' the images of A and C under the transformation by matrix N . (2 marks)
- (b) On a pair of axes given in the Answer Book draw and label the image $A'B'C'$ of this triangle. (2 marks)
- (c) Describe fully the transformation represented by matrix N . (1 mark)
- (d) Give one property of triangle ABC that is invariant under this transformation. (1 mark)

STRAND 7

STATISTICS

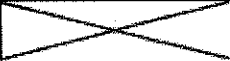
[6 marks]

- This strand has **3 Questions**
- **Choose** the best answer and write the **letter** of your choice for **Questions 1 - 2**.
- Show all working for **Question 3**.

1. A set of scores has a **mean** of 5. If all of the scores are **multiplied** by 10, the **new mean** will be
- A. 5
B. 10
C. 15
D. 50
- (1 mark)

2. A set of scores has a standard deviation of 6. If 10 is **added** to each score, the new **standard deviation** will be
- A. 6
B. 10
C. 16
D. 60
- (1 mark)

3. The table below shows scores from a maths quiz.

x	f	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
1	1	1	4	4
2	4	8	1	4
3	9	27	<input type="text"/>	<input type="text"/>
4	6	24	<input type="text"/>	<input type="text"/>
Total	20	60		14

- (a) Calculate the **mean** score. (1 mark)
- (b) Hence, complete the table by filling in the missing values in the boxes in the **Answer Booklet**. (2 marks)
- (c) Calculate the **standard deviation**. (1 mark)

STRAND 8

PROBABILITY

[10 marks]

- This strand has **5 Questions**
 - **Choose** the best answer and write the **letter** of your choice for **Questions 1 - 2**.
 - Show all working for **Questions 3 - 5**.
1. A set of quiz score is **normally distributed** with mean = 50 and standard deviation = 5. A score selected at random will **almost certainly** lie between
- A. 35 and 50
 B. 35 and 65
 C. 40 and 60
 D. 45 and 55
2. The **outcomes** of a coin and a die tossed together are shown below.

(1 mark)

		die					
		1	2	3	4	5	6
coin	H	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
	T	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

What is the probability of getting a **tail** and an **even number**?

- A. $\frac{1}{12}$
 B. $\frac{2}{12}$
 C. $\frac{3}{12}$
 D. $\frac{6}{12}$

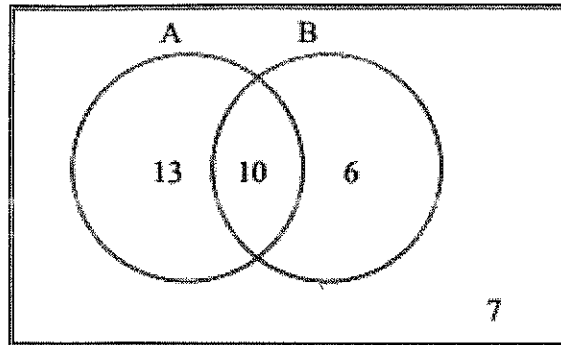
(1 mark)

3. A jar contains 3 red marbles and 2 green marbles, all of same size and shape. A marble is withdrawn at random and its colour is noted. **Without replacing** this marble, another marble is randomly withdrawn. What is the probability that the marbles are of **different** colours?

(2 marks)

4. The **Venn diagram** shown below illustrates the results of a survey of the subjects taken by a class of Year 12 students.

A = number of students taking Agriculture.
 B = number of students taking Biology.



- (a) How many students were surveyed? (1 mark)
- (b) One student is picked at random from this class. What is the **probability** that the student takes Agriculture? (1 mark)
5. The heights of Year 12 students are **normally distributed** with a mean of 160 cm and a standard deviation of 4 cm.
- (a) What is the probability that a randomly selected Year 12 student has the height **between** 149 cm and 171 cm? Give your answer correct to **3 decimal places**. (3 marks)
- (b) From a sample of 1000 Year 12 students, how many may be **expected** to have the height **between** 149 cm and 171 cm? (1 mark)

STRAND 9

CALCULUS

[18 marks]

- This strand has **6 Questions**
- **Choose** the best answer and write the **letter** of your choice for **Questions 1 - 2**.
- Show all working for **Questions 3 - 6**.

1. The derivative of a function $f(x)$ is given by $f'(x) = -3x$.
For which values of x is the function $f(x)$ **increasing**?

- A. $x < 0$
- B. $x > 0$
- C. $x \leq 0$
- D. $x \geq 0$

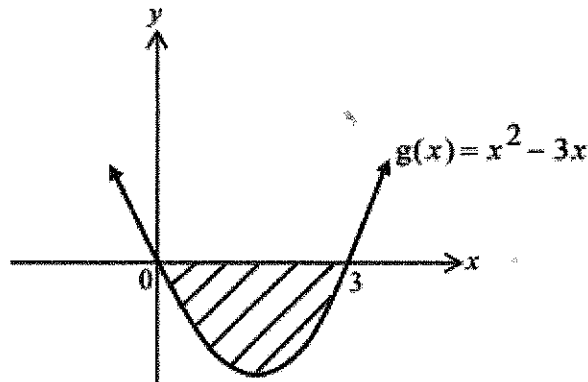
(1 mark)

2. If $y = f(x)$ then $f'(k)$, where k is a constant, is equal to the

- A. gradient of the normal at $x = k$
- B. gradient of the tangent at $x = k$
- C. definite integral between 0 and k
- D. area enclosed between $y = f(x)$ and $y = k$

(1 mark)

3. The area bounded by the curve $g(x) = x^2 - 3x$ and the x -axis is shaded below.



Calculate the **area** of the shaded region.

(3 marks)

4. A **cubic function** is given by the equation $y = x^3 + 3x^2 - 9x + 2$.

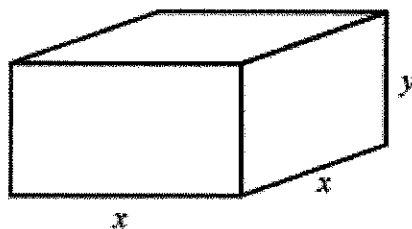
(a) **Differentiate** to find $\frac{dy}{dx}$ (2 marks)

(b) Hence, determine the **coordinates** of the turning points. (3 marks)

5. The **gradient function** of a curve is given by $\frac{dy}{dx} = x^2 - 9$.

Find the equation of the curve given that it passes through the point (0, 1) (3 marks)

6. A box with a square base and no top needs to be made from 12m^2 of metal. The length of the base of the box is x metres and its height is y metres, as shown below.



(a) Find an expression for the height y of the box in terms of x . (1 mark)

(b) Express the volume, V , of the box in terms of x . (1 mark)

(c) Determine the value of x which will **maximize** the volume of all such boxes. (3 marks)

THE END

FORMULAE

Strand 1 - Basic Mathematics

Exponents

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$; $a \neq 0$
3. $(a^m)^n = a^{mn}$
4. $(ab)^m = a^m b^m$
5. $a^0 = 1$; $a \neq 0$

Logarithms

1. $\log ab = \log a + \log b$
2. $\log \frac{a}{b} = \log a - \log b$
3. $\log x^n = n \log x$
4. $y = b^x \leftrightarrow \log_b y = x$
5. $y = e^x \leftrightarrow x = \ln y$ ($\ln y = \log_e y$)

Strand 2 - Algebra

Arithmetic Sequence

1. $T_n = a + (n-1)d$
2. $S_n = \frac{n}{2}[2a + (n-1)d]$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Geometric Sequence

1. $S_n = \frac{a(1-r^n)}{1-r}$; $r \neq 1$
2. $T_n = ar^{n-1}$
3. $S_\infty = \frac{a}{1-r}$; if $|r| < 1$

Strand 3 - Graphs

Equation of a circle centre (a, b) with radius r is

$$(x-a)^2 + (y-b)^2 = r^2$$

Strand 4 - Coordinate Geometry

1. Distance

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

3. Gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

2. Equation of the line through (x_1, y_1) and (x_2, y_2) is

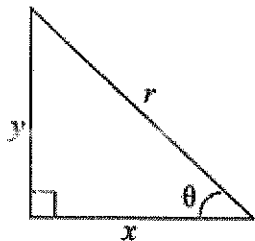
$$y - y_1 = m(x - x_1)$$

Strand 5 - Trigonometry**Area**

1. Area of triangle = $\frac{1}{2}ab \sin C$
2. Area of Sector = $\frac{1}{2}r^2\theta$

Sine and Cosine Rules

1. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
2. $a^2 = b^2 + c^2 - 2bc \cos A$

Basic Functions

1. $\sin \theta = \frac{y}{r}$
2. $\cos \theta = \frac{x}{r}$
3. $\tan \theta = \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$
4. $\cot \theta = \frac{x}{y} = \frac{\cos \theta}{\sin \theta}$
5. $\sec \theta = \frac{r}{x} = \frac{1}{\cos \theta}$
6. $\operatorname{cosec} \theta = \frac{r}{y} = \frac{1}{\sin \theta}$

Basic Identities

1. $\cos^2 \theta + \sin^2 \theta = 1$
2. $\tan^2 \theta + 1 = \sec^2 \theta$

Strand 6 - Matrices and Transformation

$$\text{If } M = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

1. $\det M$ ($|M|$) = $ad - bc$
2. $M^{-1} = \frac{1}{|M|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Strand 7 - Statistics**Individual Data**

1. $\bar{x} = \frac{\sum x}{n}$
2. $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$

Grouped Data

1. $\bar{x} = \frac{\sum fx}{\sum f}$
2. $s = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$

Strand 9 - Calculus**Differentiation**

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Indefinite Integral

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

